

MATH 648, PROBLEM SET 1

These problems are due in class on February 11.

- (1) Maclagan-Sturmfels, exercise 1.2.
- (2) Maclagan-Sturmfels, exercise 1.10.
- (3) Suppose that k is a field of characteristic not 2. Find two roots of the polynomial $x^2 - x - t^{-1}$ in the field $k\{\{t\}\}$.
- (4) Maclagan-Sturmfels, exercise 2.6(c-e).
- (5) Maclagan-Sturmfels, exercise 2.10.
- (6) This problem is canceled due to incorrectness. For the record, the previous version is as follows:

Let I be a homogeneous ideal in $K[x_0, \dots, x_n]$, and we write I_{triv} for the same ideal, but where we consider K to have the trivial valuation. Prove that there exists a constant C such that for any $w \in \Gamma^{n+1}$, there exists a $v \in \Gamma^{n+1}$ with $|v| < C$ and such that $\text{in}_{w+v}(I)$ is the reduction of $\text{in}_w(I_{\text{triv}}) \cap R[x_0, \dots, x_n]$ modulo the maximal ideal \mathfrak{m} .

This is a variant of Maclagan-Sturmfels, exercise 2.11, which is incorrect. Note when reading that problem that, somewhat confusingly the “usual initial ideal” means not only ignoring valuations, but also using the opposite sign convention. You can interpret “usual initial ideal of I with respect to $-w$ ” as “initial ideal for trivially valued field with respect to w .”

- (7) Maclagan-Sturmfels, exercise 2.14.