

Math 55 worksheet, April 1, 2009

1. Find a recurrence relation for the number of permutations of a set with n elements.
2. A string that contains only 0s, 1s, and 2s is called a *ternary string*. Find a recurrence relation for the number of ternary strings that do not contain two consecutive 0s. What are the initial conditions? How many ternary strings of length six do not contain two consecutive 0s?
3. Solve these recurrence relations together with the given initial conditions:
 - (a) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.
 - (b) $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$.
 - (c) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 1$.
 - (d) $a_n = 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 4$.
4. The *Lucas numbers* satisfy the recurrence relation $L_n = L_{n-1} + L_{n-2}$ and the initial conditions $L_0 = 2$ and $L_1 = 1$.
 - (a) Show that $L_n = f_{n-1} + f_{n+1}$ for $n = 2, 3, \dots$, where f_n is the n th Fibonacci number.
 - (b) Find an explicit formula for the Lucas numbers.
5. Suppose that X_1 and X_2 are independent Bernoulli trials each with probability $1/2$, and that $x_3 = (X_1 + X_2) \bmod 2$.
 - (a) Show that X_1 , X_2 , and X_3 are pairwise independent, but X_3 and $X_1 + X_2$ are not independent.
 - (b) Show that $V(X_1 + X_2 + X_3) = V(X_1) + V(X_2) + V(X_3)$.
6. Let X be a random variable on the sample space S such that $X(s) \geq 0$ for all $s \in S$. Show that $p(X(s) \geq a) \leq E(X)/a$ for every positive real number a . This inequality is called *Markov's Inequality*.