Math 55 worksheet, March 9, 2009

1. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

2. How many license plates consisting of three letters followed by three digits contain no letter or digit twice?

3. How many bit strings of length 10 contain at least three 1s and at least three 0s?

4. Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$, both using a combinatorial argument and by algebraic manipulation.

5. In this problem, we'll give a combinatorial proof that $a^5 - a$ is divisible by 5.

   (a) How many ways are there of assigning a colors to 5 marbles if the colors are not all the same (red-red-red-red-blue is okay, but not red-red-red-red-red)?

   (b) If the marbles are arranged in a circle, and colors are assigned as above, but each color assignment is considered the same as all rotations of itself, how many ways are there of assigning colors to the marbles?

   (c) Why does this mean that $a^5 - a$ is divisible by 5.