Math 55 worksheet, February 23, 2009

1. Give a recursive definition of:

   (a) the set of positive even integers.

   (b) the set of positive integers congruent to 2 modulo 3.

   (c) the set of polynomials with integer coefficents

2. Use induction to prove that that if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets such that $A_j \subseteq B_j$ for $j = 1, 2, \ldots, n$, then

$$\bigcup_{j=1}^{n} A_j \subseteq \bigcup_{j=1}^{n} B_j$$

3. Use induction to prove that $n^2 - 1$ is divisible by 8 whenever $n$ is an odd positive integer.

4. Show that $f_{n+1} f_{n-1} - f_n^2 = (-1)^n$ when $n$ is a positive integer, where $f_n$ is the $n$th Fibonacci number.

5. Show that $n$ lines separate the plane into $(n^2 + n + 2)/2$ regions if no two of these lines are parallel and no three pass through a common point.