Math 55 worksheet, February 4, 2009

1. Show that if a is an integer and d is a positive integer greater than 1, then the quotient and remainder obtained when a is divided by d are $\lfloor a/d \rfloor$ and $a - d\lfloor a/d \rfloor$ respectively.

2. Find a formula for the integer with smallest absolute value that is congruent to an integer a modulo m, where m is a positive integer.

3. Prove that if n is an odd integer, then $n^2 \equiv 1 \pmod 8$.

4. An ISBN consists of 10 digits $x_1 x_2 \ldots x_{10}$, chosen such that $\sum_{i=1}^{10} i x_i \equiv 0 \pmod{11}$. Why is did they choose this formula and not $\sum_{i=1}^{10} x_i \equiv 0 \pmod{11}$? Given the first 9 digits, how do you choose the 10th digit so that the the string is a valid ISBN? (Note that you might have to count 10 as a digit. It is represented by an X.)

5. Show that if n and k are positive integers, then $\lceil n/k \rceil = \lfloor (n-1)/k \rfloor + 1$.