

Math 55 worksheet, January 28, 2009

1. Determine whether each of these statements is true or false:

- (a)  $x \in \{x\}$
- (b)  $\{x\} \in \{\{x\}\}$
- (c)  $\{x\} \subseteq \{x\}$
- (d)  $\emptyset \subseteq \{x\}$
- (e)  $\{x\} \in \{x\}$
- (f)  $\emptyset \in \{x\}$

2. The symmetric difference of  $A$  and  $B$ , denoted by  $A \oplus B$  is the set containing those elements in either  $A$  or  $B$ , but not in both  $A$  and  $B$ .

Determine whether the symmetric difference is associative; that is, if  $A$ ,  $B$ , and  $C$  are sets, does it follow that  $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ ?

3. Let  $f$  be a function from  $A$  to  $B$ . Let  $S$  and  $T$  be subsets of  $A$ , and let  $U$  and  $V$  be subsets of  $B$ . Three of the following equalities are true and one is false. Prove the true statements and find a counterexample for the false one.

- (a)  $f(S \cup T) = f(S) \cup f(T)$
- (b)  $f(S \cap T) = f(S) \cap f(T)$
- (c)  $f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$
- (d)  $f^{-1}(U \cap V) = f^{-1}(U) \cap f^{-1}(V)$

4. Show that a set  $S$  is infinite if and only if there is a proper subset  $A$  of  $S$  such that there is a one-to-one correspondence between  $A$  and  $S$ .

5. Show that the polynomial function  $f: \mathbf{Z}^+ \times \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$  with  $f(m, n) = (m + n - 2)(m + n - 1)/2 + m$  is one-to-one and onto.