Math 55 worksheet, review for final, May 11, 2009

1. Use the Euclidean algorithm to find gcd(1001, 1331), gcd(1000, 5040), and gcd(123, 277).

2. Find all solutions to the system of congruences:

$$x \equiv 1 \pmod{2}$$
$$x \equiv 2 \pmod{3}$$
$$x \equiv 3 \pmod{5}$$
$$x \equiv 4 \pmod{11}$$

3. How many alignments are there between the sequence $GGGGGCCCCC$ and $GGGGGGGCCCCCCCCC$ which maximize the score $M - 2(I + D)$ where M is the number of matches, I is the number of insertions and D is the number of deletions? (Hint: This is really a counting problem, not an alignment problem)

4. Show that if the statement $P(n)$ is true for infinitely many positive integers n and $P(n+1) \rightarrow P(n)$ is true for all positive integers n, then $P(n)$ is true for all positive integers n.

5. There are six runners in a 100-meter race. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish the fastest receive gold medals, the runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)

6. Let X be the number appearing on the first die when two dice are rolled and let Y be the sum of the numbers appearing on the two dice. Show that $E(X)E(Y) \neq E(XY)$.

7. Find a, b and c such that the tropical curve defined by $a \odot x \oplus b \odot y \oplus c$ contains the points $(1, 2)$ and $(0, 0)$. Find a, b, c, d, e, and f such that the tropical quadratic curve defined by $a \odot x^2 \oplus b \odot x \odot y \oplus c \odot y^2 \oplus d \odot x \oplus e \odot y \oplus f$ contains the five points $(-1, -1)$, $(0, 2)$, $(2, 1)$, $(3, -1)$, and $(4, 2)$.