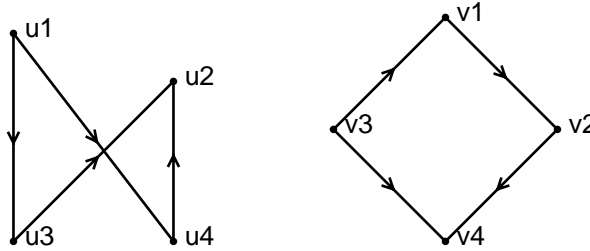


1. Show that a simple graph G is bipartite if and only if it has no circuits with an odd number of edges.
2. Define isomorphism of directed graphs.
3. Determine whether the following pair of directed graphs are isomorphic:



What is the equivalent of degree sequence for a directed graph?

4. A farmer needs to carry a wolf, a goat, and a cabbage across a river. The farmer only has a small boat, which can carry the farmer and only one object. If the farmer is on the other shore, the wolf will eat the goat, and, similarly, the goat will eat the cabbage. Make a graph where the vertices are the allowable configurations of the farmer, wolf, goat, and cabbage on the two shores, and the edges are the moves from one configuration to another by one boat trip. Find a directed path in this graph which solves the puzzle.
5. How many nonisomorphic simple graphs are there with 5 vertices and 3 edges?
6. Let $G = (V, E)$ be a simple graph. Find an equivalence relation R on the vertices V such that the equivalence classes of R are the connected components of G . Prove that R is an equivalence relation.
7. Let $G = (V, E)$ be a directed graph. Find an equivalence relation R on the vertices V such that the equivalence classes of R are the strongly connected components of G . Prove that R is an equivalence relation.
8. Show that every connected graph with n vertices has at least $n - 1$ edges.