

Math 55 worksheet, April 20, 2009

1. Which of these relations on the set of all functions from \mathbf{Z} to \mathbf{Z} are equivalence relations?
 - (a) $\{(f, g) \mid f(1) = g(1)\}$
 - (b) $\{(f, g) \mid f(0) = g(0) \text{ or } f(1) = g(1)\}$
 - (c) $\{(f, g) \mid f(x) - g(x) = 1 \text{ for all } x \in \mathbf{Z}\}$
2. Show that the lexicographic order is a partial ordering on the Cartesian product of two posets.
3. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.
4. Given an interpretation of the equivalence classes for the equivalence relation R in the previous exercise. [Hint: Look at the difference $a - b$ corresponding to (a, b) .]
5. Problem 16 on the homework asked you to show that the relation on ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Given an interpretation of the equivalence classes of this equivalence relation.
6. Let (S, \preceq) be a poset. We say that an element $y \in S$ *covers* an element $x \in S$ if $x \prec y$ and there is no element $z \in S$ such that $x \prec z \prec y$. The set of pairs (x, y) such that y covers x is called the covering relation of (S, \preceq) .

What is the covering relation of the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the power set of S where $S = \{a, b, c\}$?
7. Show that a finite poset can be reconstructed from its covering relation. [Hint: Show that the poset is the reflexive transitive closure of its covering relation.]