Math 54 worksheet, October 19, 2009

1. Let W be the vector space in $\mathbb{R}^3$ spanned by the vectors

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} \right\}.$$

   Find an orthonormal basis for W.

   Solution: An orthonormal basis is:

$$\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \right\}$$

2. Find the point in W closest to the vector

$$\begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix}$$

   Solution:

$$\begin{bmatrix} 3 \\ -3 \\ -3 \end{bmatrix}$$

3. If $\mathbf{u}$ is any vector in $\mathbb{R}^n$, show that the set $\{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{u} \cdot \mathbf{x} = 0\}$ is a subspace of $\mathbb{R}^n$.

4. Show that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is an orthogonal set of non-zero vectors in $\mathbb{R}^n$, then it is a basis for $\mathbb{R}^n$.

5. Suppose A is an orthogonal $n \times n$ matrix, i.e. the column vectors form an orthogonal set. Show that A is invertible.

6. Show that if A is an orthogonal $n \times n$ matrix, and λ is a (real) eigenvalue of A, then λ is either 1 or -1. (Hint: if $\mathbf{v}$ is an eigenvector, think about the inner product of $A\mathbf{v}$ with itself.)