Math 54 worksheet, September 30, 2009

1. Write the solutions to the following system of equations in parametric vector form:

$$\begin{aligned} 3x_1 + 2x_2 + 2x_3 &= 7 \\ 2x_1 - 2x_2 + 8x_3 &= 8 \\ x_1 + 4x_2 - 6x_3 &= -1 \end{aligned}$$

The solutions are:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

2. Find bases for the row space, column space, null space, and left null space of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & -2 & 10 \\ -1 & -4 & 6 \end{bmatrix}.$$

Indicate which of these are Schubert bases and which are reverse Schubert bases.

Reverse Schuber basis for Nul(A) is $\left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \right\}$. Schubert basis for Row$(A)$ is $\left\{ \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \right\}$. Basis for Col$(A)$ is $\left\{ \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -4 \end{bmatrix} \right\}$. Reverse Schubert basis for LeftNul(A) is $\left\{ \begin{bmatrix} -14 & 5 & 1 \end{bmatrix} \right\}$.

3. Find the inverse and compute the determinant of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The determinant is 1 and the inverse is:

$$\begin{bmatrix} 1 & -2 & -5 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation that projects each vector $\mathbf{x} = (x_1, x_2, x_3)$ onto the plane $x_2 = 0$, so $T(\mathbf{x}) = (x_1, 0, x_3)$. Show that T is a linear transformation and find its matrix.

The matrix is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Let V be the vector space of all polynomials of degree at most 3. Let W be the set of all polynomials f in V such that $f(2) = 0$. Prove that W is a subspace. Find a basis for W. Let U be the set of all polynomials f in V such that $f(2) = 1$. Is U a subspace?

   A basis for W is $\{x^3 - 8, x^2 - 4, x - 2\}$. U is not a subspace because $x - 1$ is in U, but $2(x - 1) = 2x - 2$ is not.