Math 54 worksheet, September 14, 2009

1. Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which consists of first rotating 30 degrees about the z axis and then rotating 90 degrees about the x axis. (I haven't specified the directions of the rotations. Use whichever ones you prefer.) What is the matrix for T? Can you describe T as a single rotation?

2. Let
$$A = \begin{bmatrix} -1 & 2 & 3 & 0 \\ 2 & -5 & 7 & 4 \\ 1 & -3 & 10 & 4 \end{bmatrix}$$
Find
   - A basis for Col(A)
   - A Schubert basis for Row(A)
   - A reverse Schubert basis for Null(A)
   - The dimension of LeftNull(A) (Hint: you can figure this out just from the echelon form of A. You don't need to row reduce A^T)

   Write the second row of A as a linear combination of the Schubert basis for Row(A).

3. Let A be the same as in the previous question. Is the vector
$$\begin{bmatrix} 7 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$
in Null(A)? Is the vector $\begin{bmatrix} 1 & 2 & 1 & 1 \end{bmatrix}$ in Row(A)? Is the vector space spanned by $\begin{bmatrix} 2 & -4 & -6 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & -10 & -4 \end{bmatrix}$ contained in Row(A)?

4. Let
$$B = \begin{bmatrix} -1 & 2 & -4 \\ 3 & -6 & 12 \\ 2 & -4 & 8 \end{bmatrix}$$
Find a reverse Schubert basis for LeftNull(B).

5. Let S be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which takes a vector and rotates it by 60 degrees around the axis spanned by $(1, 1, 0)$. Can you find a matrix for S?

Hermann Schubert (1848-1911)