Math 54 quiz solutions October 15, 2009

1 *Define what it means for a set of vectors to be* linearly dependent *(3 points).*

A linear combination of vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is an expression $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ where $c_1, \ldots, c_n$ are scalars.

2 *Define* row space *(3 points).*

If A is an $n \times m$ matrix, then each row of A is a vector in $\mathbb{R}^m$. The row space of the matrix A is defined to be the span of these vectors, i.e. the set of all linear combinations of the rows of A.

3 *Define the* inverse *of a matrix (3 points).*

The inverse of an $n \times n$ matrix A is an $n \times n$ matrix, written A^{-1} such that $AA^{-1} = A^{-1}A = I_n$, where I_n is the $n \times n$ identity matrix.

4 *Define* left nullspace *(3 points).*

The *left nullspace* of an $n \times m$ matrix A is the set of all vectors $\mathbf{x}$ in $\mathbb{R}^n$ such that $\mathbf{x}^T A = 0$.

5 *Define* linear subspace *(3 points).*

A subset V of a vector space is a linear subspace if the following are all true:

1. The vector $\mathbf{0}$ is in V.
2. If $\mathbf{v}$ and $\mathbf{w}$ are in V, then $\mathbf{v} + \mathbf{w}$ is in V.
3. If $\mathbf{v}$ is in V and c is a real number, then $c\mathbf{v}$ is in V.

6 *Show that if the span of* $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ *is all of* $\mathbb{R}^n$*, and* A *is an invertible matrix, then the span of* $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$ *is also all of* $\mathbb{R}^n$ *(5 points).*

Let $\mathbf{v}$ be any vector in $\mathbb{R}^n$. Since $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ spans $\mathbb{R}^n$, $A^{-1}\mathbf{v}$ can be written as $A^{-1}\mathbf{v} = c_1\mathbf{v}_1 + \cdots c_n\mathbf{v}_n$ for some scalars $c_1, \ldots, c_n$. Multiplying this equation by A, we have the equations:

$$\begin{aligned} AA^{-1}\mathbf{v} &= A(c_1\mathbf{v}_1 + \cdots c_n v_n) \\ \mathbf{v} &= c_1 A\mathbf{v}_1 + \cdots c_n A\mathbf{v}_n. \end{aligned}$$

Thus, $\mathbf{v}$ is in the span of $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$. Therefore, the span of these vectors is all of $\mathbb{R}^n$.