1 *Find the general solution to the differential equation (5 points)*

$$y'' + y' - 2y = e^{-2t}$$

The auxillary equation of the homogeneous differential equation is $r^2 + r - 2 = (r+2)(r-1)$ so the roots are $r = -2$ and $r = 1$. Thus, the general solution to the homogeneous equation is $Ae^{-2t} + Be^t$.

Our guess for a solution to the inhomogeneous equation would be ae^{-2t}, but this is a solution to the homogeneous equation. Thus, we try $y = ate^{-2t}$ where a is an unknown coefficient. The derivatives are

$$\begin{aligned} y' &= ae^{-2t} - 2ate^{-2t}, \\ y'' &= -2ae^{-2t} - 2ae^{-2t} + 4ate^{-2t} = -4ae^{-2t} + 4ate^{-2t}. \end{aligned}$$

Substituting this into the differential equation we get:

$$\begin{aligned} y'' + y' - 2y &= (-4ae^{-2t} + 4ate^{-2t}) + (ae^{-2t} - 2ate^{-2t}) - 2(ate^{-2t}) \\ &= -3ae^{-2t}. \end{aligned}$$

If this is equal to the right hand side, e^{-2t}, then $a = -1/3$, so the general solution is

$$y = Ae^{-2t} + Be^t - \tfrac{1}{3}te^{-2t}$$

2 *Find a solution to the differential equation*

$$y'' + 2y' + 2y = 0$$

with initial conditions $y(0) = 2$ *and* $y'(0) = 1$ *(5 points).*

The auxillary equation is $r^2 + 2r + 2$. Using the quadratic formula, the roots are:

$$r = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2}}{2} = -1 \pm \frac{\sqrt{-4}}{2} = -1 \pm i$$

Thus, the general solution is $y = Ae^{-t}\cos t + Be^{-t}\sin t$.

The derivative is

$$y' = -Ae^{-t}\cos t - Ae^{-t}\sin t - Be^{-t}\sin t + Be^{-t}\cos t.$$

Evaluating at $t = 0$, we have

$$y(0) = A \quad \text{and} \quad y'(0) = -A + B.$$

Our initial conditions tell us that $A = 2$ and $-A + B = 1$, so $B = 3$. Therefore, the solution is $y(t) = 2e^{-t}\cos t + 3e^{-t}\sin t$.