1 *Use Cramer's rule to find the value of x_3 in the solution to the equation:*

$$\begin{bmatrix} 3 & -2 & -5 \\ -1 & -1 & 3 \\ 6 & 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Call the matrix on the left hand side A. We compute the determinant of A by expanding by minors along the first column:

$$\begin{aligned} A &= 3\begin{vmatrix} -1 & 3 \\ 1 & -6 \end{vmatrix} + 1\begin{vmatrix} -2 & -5 \\ 1 & -6 \end{vmatrix} + 6\begin{vmatrix} -2 & -5 \\ -1 & 3 \end{vmatrix} \\ &= 3(6-3) + (12+5) + 6(-6-5) \\ &= 9 + 17 - 66 = -40 \end{aligned}$$

Then we substitue the right hand side into the third column of A and compute its determinant, again by expanding along the first column:

$$\begin{aligned} \begin{vmatrix} 3 & -2 & 2 \\ -1 & -1 & -1 \\ 6 & 1 & 3 \end{vmatrix} &= 3\begin{vmatrix} -1 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} -2 & 2 \\ 1 & 3 \end{vmatrix} + 6\begin{vmatrix} -2 & 2 \\ -1 & -1 \end{vmatrix} \\ &= 3(-3+1) + (-6-2) + 6(2+2) \\ &= -6 - 8 + 24 = 10 \end{aligned}$$

Thus, the value of x_3 is $10/(-40) = -1/4$. (In fact, the solution is $(1/4, 0, -1/4)$.)

2 *Find the inverse of the matrix:*

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 2 & 2 \\ 3 & 0 & -1 \end{bmatrix}$$

We put the identity matrix to the right of A and row reduce:

$$\begin{bmatrix} 2 & 3 & 3 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 3 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 0 & 1 & 0 \\ 2 & 3 & 3 & 1 & 0 & 0 \\ 3 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 0 \\ 0 & -6 & -7 & 0 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & -6 & -7 & 0 & -3 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & -3 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & 0 & -1 & -6 & 9 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 & -3 & 0 \\ 0 & 1 & 0 & -7 & 11 & 1 \\ 0 & 0 & -1 & -6 & 9 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & -3 & 0 \\ 0 & 1 & 0 & -7 & 11 & 1 \\ 0 & 0 & 1 & 6 & -9 & -1 \end{bmatrix}$$

Therefore, the inverse of A is:

$$\begin{bmatrix} 2 & -3 & 0 \\ -7 & 11 & 1 \\ 6 & -9 & -1 \end{bmatrix}$$