Math 54 quiz solutions December 2, 2009

1 *Find the Fourier cosine series expansion of* $f(x) = 3 + x$ *defined on* $0 < x < \pi$ *(7 points).*

For $n > 0$, we compute a_n using the formula:

$$a_n = \frac{2}{\pi} \int_0^\pi (3 + x) \cos nx \, dx$$

Use integration by parts with $u = 3 + x$, $dv = \cos nx$, $du = dx$, $v = \frac{1}{n} \sin nx$:

$$\begin{aligned} &= \frac{2}{\pi} \left[\frac{3 + x}{n} \sin nx \right]_{x=0}^{\pi} - \frac{2}{\pi} \int_0^\pi \frac{1}{n} \sin nx \, dx \\ &= 0 + \frac{2}{\pi} \left[\frac{1}{n^2} \cos nx \right]_{x=0}^{\pi} \end{aligned}$$

because $\sin n\pi = \sin 0 = 0$ for any integer n,

$$= \frac{2}{n^2 \pi} \left((-1)^n - 1 \right).$$

This is $\frac{-4}{n^2\pi}$ for n odd and 0 for n even.

To compute a_0,

$$a_0 = \frac{2}{\pi} \int_0^\pi 3 + x \, dx = \frac{2}{\pi} \left[3x + \frac{x^2}{2} \right]_{x=0}^{\pi} = \frac{2}{\pi} \left(3\pi + \frac{\pi^2}{2} \right) = 6 + \pi$$

Thus, the Fourier series is:

$$\left(3 + \frac{\pi}{2} \right) + \sum_{k=1}^{\infty} \frac{-4}{(2k - 1)^2 \pi} \cos \left((2k - 1)x \right)$$

2 *What is the value of the Fourier series from the previous problem for* x *in the range* $-\pi < x < 0$ *(3 points)?*

Since the cosine is an even function, a Fourier cosine series always gives an even function. Thus, for $-\pi < x < 0$, the Fourier cosine series converges to $f(-x) = 3 - x$.