MATH 380A/500A, PROBLEM SET 9

These problems are due at the beginning of class on Monday, Nov. 11.

(1) Eisenbud, exercise 11.5. In particular, note that the principal ideal generated by $x^{(0,1)}$ in this ring is prime and has codimension greater than 1.

(2) Let R be a local domain, not assumed to be Noetherian. Suppose that the maximal ideal $\mathfrak{m}$ is principal and $\cap_{n=1}^{\infty} \mathfrak{m}^n = 0$. Show that R is a discrete valuation ring.

(3) Eisenbud, exercise 11.10.

(4) Let R be a Dedekind domain and let M be a finitely generated torsion module over R, i.e. for each $m \in M$, there exists non-zero $r \in R$ such that $rm = 0$. Prove that $M \cong \oplus R/\mathfrak{m}_i^{e_i}$ where the $\mathfrak{m}_i$ are maximal and the e_i are positive integers. (Hint: localize and use the structure theorem for principal ideal domains.)

(5) Let k be a field an suppose R is a Dedekind domain which is finitely generated as ring over k. Prove that if $I \subset R$ is a non-zero ideal, $\dim_k R/I$ is finite and that the function $I \mapsto \dim_k R/I$ factors through $C(R)$.

(6) Show that every ideal in a Dedekind domain R can be generated by at most 2 elements.