MATH 380A/500A, PROBLEM SET 8

These problems are due at the beginning of class on Monday, Nov. 4.

(1) Eisenbud, exercise 9.1.
(2) Eisenbud, exercise 10.1.
(3) Eisenbud, exercise 10.2.
(4) Let k be a field. Show that the ring $k[x] \times k[x]$ is regular, but is not a domain.
(5) Eisenbud, exercise 10.5. You don't need to do anything for the sentence beginning "Note that..."
(6) Eisenbud, exercise 10.8. Figure 10.6 is an illustration of the ideal $\langle xy, xz\rangle = \langle x\rangle \cap \langle y, z\rangle$.
(7) Eisenbud, exercise 11.1.
(8) (Correction of Eisenbud, exercise 11.4) Let G be any ordered Abelian group, as described in the exercise. Let k be a field and R the vector space over k with bases $\{x^a \mid a \in G\}$. Define a multiplication operation on R by $x^a x^b = x^{a+b}$ and using linearity. Show that the resulting ring R is a domain, and that its field of fractions has a well-defined function $\nu\colon K(R)^* \to G$ by:
$$\nu\left(\sum r_a x^a\right) = \min\{a \mid r_a \neq 0\}$$
for $r_a \in k$, and
$$\nu(r/s) = \nu(r) - \nu(s)$$
for $r, s \in R$. Show that this ν satisfies the axioms of a valuation on page 252.