MATH 380A/500A, PROBLEM SET 7

These problems are due at the beginning of class on Monday, October 28.

(1) Eisenbud, exercise 7.2.
(2) Suppose R is complete with respect to an ideal I. Show that I is contained in the Jacobson radical of R.
(3) Eisenbud, exercise 7.11.
(4) Eisenbud, exercise 7.12.
(5) Eisenbud, exercise 7.16. For the last sentence, it's important that you not just find the coefficient field of $\widehat{R}$, but describe it as a subring of $\widehat{R}$.
(6) Eisenbud, exercise 7.27. Please note hint in back of the book.
(7) Let $\alpha_n \colon \mathbb{Z}/p \to \mathbb{Z}/p^n$ be the injective morphism of $\mathbb{Z}$-modules defined by $\alpha_n(1) = p^{n-1}$. Let $\alpha \colon A \to B$ be the direct sum of the α_n for $n \geq 1$, i.e. $A = \bigoplus_{n=1}^{\infty} \mathbb{Z}/p$ and $B = \bigoplus_{n=1}^{\infty} \mathbb{Z}/p^n$ and α is defined component-wise. Prove that the p-adic completion of A is A, but that the completion of A with respect to the topology induced by the p-adic topology on B is infinite direct product $\prod_{n=1}^{\infty} \mathbb{Z}/p$.
(8) Eisenbud, exercise 9.6.