MATH 380A/500A, PROBLEM SET 10

These problems are due in class on November 20.

(1) Eisenbud, exercise 12.3.

To clarify the hint in Eisenbud, the module structure on both sides of the surjection is as a $(R/\mathfrak{q})[x_1, \ldots, x_d]$-module. On $\operatorname{gr}_{\mathfrak{q}} M$, the variables x_i act like the images of the generators of $\mathfrak{q}$. The first sentence of the hint is sufficient for the first sentence of the exercise and the second sentence of the hint is only relevant for the second sentence of the exercise.

(2) Eisenbud, exercise 1.21b.

(3) Eisenbud, exercise 12.11b. The multiplicity of a module is defined in Exercise 12.6.

(4) Eisenbud, exercise 12.12.

Note that part b is independent of commutative algebra, but is just a fact about the power series expansion of rational whose denominator is of the form $\prod(1 - t^{d_i})$

(5) Eisenbud, exercise 13.1.

(6) Eisenbud, exercise 13.6.

(7) Eisenbud, exercise 13.9.

You want to imitate the structure of the proof of polynomial Noether normalization, beginning with an analogue of Lemma 13.2. As stated in the exercise, you want to use Exercises 7.2 and 7.4 (you can use these without proof), and I believe it will be sufficient to take the x_i' to be a subset of the variables x_i. Then the proof is similar to the polynomial case.

(8) None. Exercise 13.10 has been delayed to the next problem set.