Math 1B Problems, volume 4

Dustin Cartwright[1]

1. Solve $y'' - 4y = e^x \cos x$ using the method of undetermined coefficients with the initial conditions $y(0) = 1$ and $y'(0) = 2$.
2. Solve $y'' + 3y' + 2y = \sin(e^x)$.
3. Solve $y'' - 2y' + y = \dfrac{e^x}{1+x^2}$.
4. Find the general solution to $y'' - 8y' + 16y = e^{4x}$ using the method of undetermined coefficients and using variation of parameters.
5. Solve $y'' + y = 3\sin 2x + x\cos 2x$.
6. Use the following recursion formulas to find explicit formulas:
   (a) $a_{n+1} = a_n + 5$ for $n \geq 0$
   (b) $a_{n+2} = a_n + 5$ for $n \geq 0$
   (c) $a_{n+1} = na_n$ for $n \geq 1$
   (d) $a_{n+1} = n^2 a_n$ for $n \geq 1$
7. Find an explicit formula for $a_{n+1} = \frac{k-n}{n+1} a_n$.
8. Solve $y' = \frac{ky}{1+x}$ using power series and explicitly. Did you get the same answer?
9. Find a solution to $x^2y'' + xy' + (x^2 - 1)y = 0$ with $y(0) = 0$ and $y(1) = \frac{1}{2}$. Try to simplify as much as you can.

[1] Many problems borrowed from various sources, mostly the Math 1b workbook