

Math 1B Problems, volume 2
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1. Which of the following are p -series or can be treated as p -series? Which are geometric series? If it is a p -series or geometric series, say whether it converges.

(a) $\sum_{n=0}^{\infty} 3^{-n}$

(b) $1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \dots$

(c) $\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{n^3}$

(d) $32 + 16 + 8 + 4 + \dots$

(e) $\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$

(f) $\sum_{n=5}^{\infty} \frac{1}{\sqrt[3]{n-3}}$

(g) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n^2}$

2. Suppose $\sum_{n=1}^{\infty} a_n$ is an infinite series. Write $\{s_n\}$ for the sequence of partial sums. Which of the following are possible or impossible:

(a) $\sum_{n=1}^{\infty} a_n$ diverges, but $\{s_n\}$ and $\{a_n\}$ converge.

(b) $\sum_{n=1}^{\infty} a_n$ and $\{s_n\}$ diverge, but $\{a_n\}$ converges.

(c) $\sum_{n=1}^{\infty} a_n$ and $\{s_n\}$ converge, but $\{a_n\}$ diverges.

(d) $\sum_{n=1}^{\infty} a_n$, $\{s_n\}$, and $\{a_n\}$ all converge.

(e) $\sum_{n=1}^{\infty} a_n$, $\{s_n\}$, and $\{a_n\}$ all diverge.

3. A student (probably the same one who tried to integrate $\int 1/x dx$ and got $0 = 1$) tries to find the value of $\sum_{n=0}^{\infty} (-1)^n$ and, once again, gets $0 = 1$.

$$\begin{aligned}\sum_{n=0}^{\infty} (-1)^n &= 1 - 1 + 1 - 1 + \dots \\ &= (1 - 1) + (1 - 1) + \dots \\ &= 0\end{aligned}$$

$$\begin{aligned}\sum_{n=0}^{\infty} (-1)^n &= 1 - 1 + 1 - 1 + \dots \\ &= 1 + (-1 + 1) + (-1 + 1) + \dots \\ &= 1\end{aligned}$$

¹Problems borrowed from various sources, mostly the Math 1b workbook

The student thinks that this manipulation is the same as what he would do with a telescoping series. Why is this not like a telescoping series? (Hint: think about the partial sums)

4. Explain why it is necessary that the function f in the Integral Test be positive and decreasing. Does f have to be *always* decreasing or is it enough for f to be *eventually* decreasing?
5. Let c be a positive constant and find $\int_0^{\infty} ce^{-cx} dx$
6. Find $\int e^x \sin 2x dx$.
7. Find $\int \frac{2x^2 - 2x - 2}{x^3 + 2x^2 + 2x} dx$.
8. Find $\int \frac{x^3}{\sqrt[3]{x^2 + 1}} dx$.
9. Find $\int \sin^2 x dx$. Find $\int \sin^3 x dx$. Let n be a positive integer. Find $\int \sin^n x dx$.
10. Find $\int \frac{dx}{1 + e^x}$.
11. If A is a real number, find $\int \frac{A}{x^2 + x + 1} dx$.
12. Find the arclength of $f(x) = |x| \cdot x$ between $(-1, -1)$ and $(1, 1)$.