Math 1B Problems
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1. If f is a continuous function on $[a, b]$ and
$$g(x) = \int_x^b f(t)\, dt$$
What is $g'(x)$?

2. Find a function f and a number a such that
$$4 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$$
for all $x > 0$.

3. Find the value of
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$$
(Hint: The sum is actually a Riemann sum for a function defined on $[0, 1]$).

4. Prove that
$$\int_{-1}^{1} \sin(x^3) + \cos(x^3)\, dx \leq 2$$
Can you find a lower bound for $\int_{-1}^{1} \sin(x^3) + \cos(x^3)\, dx$?

5. Compute the following integrals:
$$\int x\cos(x)\, dx$$
$$\int \ln x\, dx$$
$$\int x^2 e^{2x}\, dx$$
$$\int e^x \sin(2x)\, dx$$
$$\int \frac{\ln(x)}{x}\, dx$$

6. What is wrong? One student uses integration by parts on the integral $\int 1/x\, dx$ as below and comes to the conclusion that $0 = 1$. Find out which step is wrong:

   (a) Let $u = 1/x$, $dv = dx$, $du = -1/x^2\, dx$, $v = x$

[1]Problems borrowed from various sources, mostly the Math 1b workbook

(b) $\int (1/x)\,dx = (1/x)x - \int x(-1/x^2)\,dx = 1 + \int (1/x)\,dx$

(c) Subtracting $\int (1/x)\,dx$ from both sides we get $1 = 0$.

7. Define

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

Find $\int \operatorname{erf}(x)\,dx$ (Hint: the answer will itself involve $\operatorname{erf}(x)$).

8. (a) Prove the reduction formula

$$\int (\ln(x))^n dx = x(\ln(x))^n - n \int (\ln x)^{n-1}\,dx$$

(b) Evaluate $\int (\ln x)^3\,dx$.

9. What is the *structure* of the partial fractions decomposition for each of the following integrals? Don't bother finding the actual decomposition; leave the coefficients undermined. For example:

$$\int \frac{2}{1-x^2}\,dx = \int \left(\frac{A}{1-x} + \frac{B}{1+x}\right)\,dx$$

(a) $\int \frac{x^5}{(x^2-4)(x^2+3)^2}\,dx$

(b) $\int \frac{1}{x^3+2x^2+4x+8}\,dx$

10. Evaluate $\int \frac{3e^{2t}}{e^{2t}-e^t-6}\,dt$.

11. Find at least three ways to solve $\int \sin x \cos x\,dx$. Are the answers the same? Why or why not?

12. Find a substitution to turn $\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$ into a rational function.

13. Solve $\int \frac{dx}{\sqrt{1+e^x}}$ (Hint: use a substitution to get a rational function).

14. Evaluate $\int \frac{3e^{2t}}{e^{2t}-e^t-6}\,dt$.

15. Find at least three ways to solve $\int \sin x \cos x\,dx$. Are the answers the same?

16. Find a substitution to turn $\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$ into a rational function.

17. Turn $\int \frac{dx}{\sec x + \csc x}$ into an integral of a rational function.

18. If $y = f(x)$ is a function and $x = g(y)$ is its inverse, then there are two possible formulas for finding the arc length between $(a, f(a))$ and $(b, f(b))$:

$$\int_a^b \sqrt{1 + (f'(x))^2}\, dx = \int_{f(a)}^{f(b)} \sqrt{1 + (g'(y))^2}\, dy$$

Why do these equations give the same answer? Try to explain both using pictures and using a substitution.

19. For what values of p is $\int_0^1 x^p\, dx$ convergent? How about $\int_1^\infty x^p\, dx$? How about $\int_0^\infty x^p\, dx$?

20. Sketch the graph of $y = x \sin\left(\frac{1}{x}\right)$ for $0 < x \leq 1$. Is the arclength of this graph finite or infinite?