1 *Find the power series for* $\frac{1}{(x+2)^2}$ *(2 points) and simplify it to the point that it doesn't use* $\binom{k}{n}$*, factorials (!), or products of the form* $1 \cdot 2 \cdots n$ *(3 points).*

We have to use a little bit of algebra to get this into the form $(1+x)^k$, and then use the binomial expansion:

$$\begin{aligned}
\frac{1}{(x+2)^2} &= \frac{1}{4(x/2+1)^2} \\
&= \frac{1}{4}\sum_{n=0}^{\infty} \binom{-2}{n} \left(\frac{x}{2}\right)^n \\
&= \sum_{n=0}^{\infty} \frac{1}{4} \frac{(-2)(-3)\cdots(-2-n+1)}{1\cdot 2\cdots n} \frac{1}{2^n} x^n \\
&= \sum_{n=0}^{\infty} \frac{1}{2^{n+2}} (-1)^n \frac{2\cdot 3\cdots(n+1)}{1\cdot 2\cdots n} x^n \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)}{2^{n+2}} x^n
\end{aligned}$$

2 *What are the first two non-zero terms of* $\frac{1}{D+d} - \frac{1}{D-d}$ *expanded as a power series in* d/D *(5 points).*

Rearrange and then use the geometric series (or the binomial series):

$$\begin{aligned}
\frac{1}{D+d} - \frac{1}{D-d} &= \frac{1}{D(1+d/D)} - \frac{1}{D(1-d/D)} \\
&= \frac{1}{D}\sum_{n=0}^{\infty} \left(\frac{-d}{D}\right)^n - \frac{1}{D}\sum_{n=0}^{\infty} \left(\frac{d}{D}\right)^n \\
&= \frac{1}{D}\sum_{n=0}^{\infty} ((-1)^n - 1) \left(\frac{d}{D}\right)^n \\
&= -\frac{2}{D}\sum_{n=0}^{\infty} \left(\frac{d}{D}\right)^{2n+1}
\end{aligned}$$

Because $(-1)^n - 1$ is just 0 when n is even and -2 when n is odd. Thus, we only need to add up the even terms. The first two non-zero terms are:

$$-\frac{2d}{D^2} - \frac{2d^3}{D^4} + \cdots$$

It is also possible to find the least common denominator and then use the binomial series.

3 *Check that the family* $y = \frac{\sqrt{2}}{x+C}$ *satisfies the differential equation* $y'' = y^3$ *(5 points).*

Using the power rule, the derivatives of y are:

$$y' = \frac{-\sqrt{2}}{(x+C)^2}$$
$$y'' = \frac{2\sqrt{2}}{(x+C)^3}$$

On the other hand y^3 is:

$$\frac{(\sqrt{2})^3}{(x+C)^3} = \frac{2\sqrt{2}}{(x+C)^3}$$

so the differential equation holds.