1 *Find the power series for* $\sqrt[3]{1-x}$ *(3 points). Write out the first 3 non-zero terms (2 points).*

Using the binomial series, we get:

$$\begin{aligned}\sqrt[3]{1-x} &= (1-x)^{1/3} \\ &= \sum_{n=0}^{\infty} \binom{1/3}{n} (-x)^n \\ &= \sum_{n=0}^{\infty} (-1)^n \binom{1/3}{n} x^n \\ &= 1 - \frac{1/3}{1!}x + \frac{(1/3)(1/3-1)}{2!}x^2 + \cdots \\ &= 1 - \frac{x}{3} - \frac{x^2}{9} + \cdots\end{aligned}$$

2 *Estimate the error in the approximation* $\sin x \approx x$ *when* $x = 0.1$ *(5 points).*

The easiest way to do this is to notice that the Taylor series for $\sin x$ is an alternating series. Furthermore, the terms are decreasing in magnitude (when $x = 0.1$) and their limit is 0, so we can use the Alternating Series Error Test, which says that the error is less than the first term which is not part of the partial sum. In this case, the error is no greater than

$$\frac{x^3}{3!} = \frac{0.001}{6} = 0.0001\overline{6}$$

Another possible solution is to use Taylor's Inequality. In this case, the error is no greater than $\frac{M}{2!}x^2$ where M is an upper bound for $|f^{(2)}(z)|$ on the range $0 \leq z \leq 0.1$. Since $f^{(2)}(z) = -\sin z$, $|f^{(2)}(z)| \leq |z|$ and so we can choose $M = 0.1$. Thus, the error in the approximation is no greater than:

$$\frac{M}{2!}x^2 = \frac{0.1}{2!}(0.1)^2 = 0.001/2 = 0.0005$$

3 *Check that all members of the family* $y = e^{-x^2/2+C}$ *satisfy the differential equation* $y' = -xy$ *(5 points).*

Take the derivative using the chain rule:

$$\begin{aligned}y' &= e^{-x^2/2+C}(-x) \\ &= -xy\end{aligned}$$