1 *Find a power series representation for* $\frac{1}{1+x^2}$ *around* $x = 0$. *(Hint: Use a power series you already know) (5 points)*

This is similar to the expression for the geometric series, so

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$$

2 *Show that*

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

is a solution to the differential equation $f(x) + f'(x) = 0$ *(5 points)*

Taking the derivative term by term,

$$\begin{aligned} f'(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n n x^{n-1}}{n!} \\ &= \sum_{n=1}^{\infty} \frac{(-1)^n x^{n-1}}{(n-1)!} \end{aligned}$$

because the $n = 0$ term is just 0

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!}$$

Then,

$$\begin{aligned} f(x) + f'(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{((-1)^n - (-1)^n)\, x^n}{n!} = 0 \end{aligned}$$

3 *What is the function represented by the power series in the previous problem?*

$$f(x) = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = e^{-x}$$