Math 1B quiz solutions October 18, 2006

1 Find the radius of convergence and the interval of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(3n)!}$.
(5 points)

Use the ratio test:

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{x^{n+1}/(3(n+1))!}{x^n/(3n)}\right| \\
&= \lim_{n\to\infty}\frac{|x|(3n)!}{(3n+3)!} \\
&= \lim_{n\to\infty}\frac{|x|}{(3n+1)(3n+2)(3n+3)} \\
&= 0
\end{aligned}$$

For any value of x. Thus the radius of convergence is ∞. The interval of convergence is $(-\infty, \infty)$.

2 Evaluate $\displaystyle\int \frac{1}{\sqrt{8-2x-x^2}}\,dx$ (5 points)

First complete the square in the denominator: $8 - 2x - x^2 = 9 - (x+1)^2$
Then use the substitution $u = x + 1$, $du = dx$ to get:

$$\int \frac{du}{\sqrt{9-u^2}}$$

Use the substitution $u = 3\sin\theta$, $du = 3\cos\theta\,d\theta$ to get:

$$\begin{aligned}
\int \frac{1}{\sqrt{9-9\sin^2\theta}}\,3\cos\theta\,d\theta &= \int \frac{3\cos\theta}{\sqrt{9\cos^2\theta}}\,d\theta \\
&= \int \frac{3\cos\theta}{3\cos\theta}\,d\theta \\
&= \int d\theta = \theta + C = \arcsin\left(\frac{u}{3}\right) + C \\
&= \arcsin\left(\frac{x+1}{3}\right) + C
\end{aligned}$$