Math 1B quiz solutions November 15, 2006

1 *Find the general solution to* $\frac{y'}{x} = y + e^{x^2}$. *(5 points)*

First, we rearrange to get the standard form for a linear differential equation:

$$y' - yx = xe^{x^2}$$

Now we find the integrating factor:

$$e^{\int -x\,dx} = e^{-x^2/2}$$

Then we multiply our original equation by the integrating factor:

$$\begin{aligned} e^{-x^2/2}y' - e^{-x^2/2}yx &= e^{-x^2/2}xe^{x^2} \\ (e^{-x^2/2}y)' &= e^{x^2/2}x \\ e^{-x^2/2}y &= \int e^{x^2/2}x\,dx \end{aligned}$$

Use the substitution $u = x^2/2$, $du = x\,dx$:

$$\begin{aligned} &= \int e^u\,du \\ &= e^u + C \\ &= e^{x^2/2} + C \\ y &= e^{x^2} + Ce^{x^2/2} \end{aligned}$$

2 *Find the general solution to the differential equation* $y'' + y' = 12y$. *(5 points)*

The auxillary equation is $r^2 + r - 12 = 0$. This polynomial factors as $(r + 4)(r - 3)$, so it has roots at $r = -4$ and $r = 3$. Thus, the general solution to the differential equation is $C_1e^{-4x} + C_2e^{3x}$.

3 *Find the Maclaurin series for* $x \sin x^2$. *(5 points)*

The Maclaurin series for $\sin x$ is $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ Thus,

$$\begin{aligned} x\sin x^2 &= x\sum_{n=0}^{\infty} \frac{(x^2)^{2n+1}}{(2n+1)!} \\ &= \sum_{n=0}^{\infty} x\frac{x^{4n+2}}{(2n+1)!} \\ &= \sum_{n=0}^{\infty} \frac{x^{4n+3}}{(2n+1)!} \end{aligned}$$