Math 1B quiz solutions November 8, 2006

1 *Sketch a direction field for the differential equation $y' = -x/y$ (3 points) and sketch a solution (2 points). (Note that the differential equation is not defined for $y = 0$).*

The solution curve is a half-circle centered at the origin. The slopes of the slope field are all tangent to a circle centered at the origin.

2 *Find the general solution to the differential equation $y' = xy^2 + x$. (5 points)*

We can divide both sides by $y^2 + 1$ to get:

$$\frac{1}{y^2+1}y' = x$$

Thus, we have separated the two variables, so we get the integrals:

$$\int \frac{1}{y^2+1}\,dy = \int x\,dx$$
$$\arctan y + C_1 = \frac{x^2}{2} + C_2$$

Consolidating the constants by using $C = C_2 - C_1$:

$$\arctan y = \frac{x^2}{2} + C$$
$$y = \tan\left(\frac{x^2}{2} + C\right)$$

3 *The population of rockfish is modeled by the differential equation $P' = kP(1 - P/K)$ where the carrying capacity K is 10^5, $k = .1$, and the time is measured in years. If the population is 10^3 at $t = 0$, what is the equation for the number of rockfish after t years? (5 points)*

The differential equation is a logisitic equation and the solution is

$$P(t) = \frac{K}{1 + Ae^{-kt}} = \frac{10^5}{1 + Ae^{-.1t}}$$

By using the initial condition, we get that:

$$10^3 = \frac{10^5}{1+A}$$
$$1 + A = 10^2 = 100$$
$$A = 99$$

so

$$P(t) = \frac{10^5}{1 + 99e^{-.1t}}$$