Math 1B quiz solutions November 8, 2006

1 *If* $y' = F(x, y)$ *is a differential equation,* $y_0 = y(x_0)$ *is an initial condition, and the step size is* h*, what is the next point produced by Euler's method? (Hint: the* x *coordinate is* $x_0 + h$*) (5 points)*

The point $(x_0 + h, y_0 + hF(x_0, y_0))$.

2 *A population of archaea grows proportionally to the size of the population. Every five hours the population triples. If the initial population is* 1*, what is the formula for the number of archaea after* t *hours? (3 points) What is the differential equation describing the general population curve? (2 points)*

The population equation is $P = 3^{t/5}$. We can rewrite this equation as $P = e^{t(\ln 3)/5}$, which is an exponential curve with $k = (\ln 3)/5$. The differential equation describing the population curve is $P' = P(\ln 3)/5$.

3 *Find the solution to the differential equation* $y' = (1 + x^2)/y$ *with the initial condition* $y = 2$ *at* $x = 1$*. (5 points)*

Rearrange to separate the variables:

$$yy' = 1 + x^2$$

Then we get the integrals:

$$\int y\,dy = \int 1 + x^2\,dx$$

$$\frac{y^2}{2} + C_1 = x + \frac{x^3}{3} + C_2$$

$$y^2 = 2x + \frac{2x^3}{3} + 2C_2 - 2C_1$$

Use $C = 2C_2 - 2C_1$:

$$y^2 = 2x + \frac{2x^3}{3} + C$$

$$y = \sqrt{2x + \frac{2x^3}{3} + C}$$

Now we plug in $x = 1$ and $y = 2$ to get:

$$2 = \sqrt{2 + \frac{2}{3} + C}$$

$$4 = \frac{8}{3} + C$$

$$\frac{4}{3} = C$$

Thus, the solution is

$$y = \sqrt{2x + \frac{2x^3}{3} + \frac{4}{3}}$$