1. Find $\int_0^2 \sqrt{x^2+4}\,dx$.

2. Find $\int_1^e (\ln x)^2\,dx$.

3. Find $\int_0^{1/2} \frac{\arctan x}{x}\,dx$.

4. If the curve $y = \sqrt{2x - x^2}$, $0 \le x \le 1$ is rotated about the x-axis, find the area of the resulting surface.

5. Describe how one can compute $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ to within 0.01. (You do not need to carry out the computation, but if the answer involves, say, the nth partial sum, then you should say what n is.)

6. Determine whether the following converge absolutely, diverge or converge absolutely:

   (a) $\sum_{n=1}^{\infty} \frac{n-1}{n^2\sqrt{n+1}}$

   (b) $\frac{2}{1} - \frac{1}{2} - \frac{1}{3} + \frac{2}{4} - \frac{1}{5} - \frac{1}{6} + \frac{2}{7} - \dots$

   (c) $\sum_{n=1}^{\infty} \left(\arctan\left(7 + \frac{1}{n}\right) - \arctan 7\right)$

   (d) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$

   (e) $\sum_{n=1}^{\infty} \frac{1}{2n^2 - \sqrt{n}}$

   (f) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

   (g) The series $\sum_{n=1}^{\infty} a_n$ where

   $$a_n = \begin{cases} \frac{1}{n+\sqrt{n}} & \text{if } n \text{ is odd} \\ -\frac{1}{n} & \text{if } n \text{ is even} \end{cases}$$

   (h) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

   (i) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{2n+3}\right)^n$

(j) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(k) $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$

(l) $\sum_{n=1}^{\infty} (-1)^n$

(m) $\sum_{n=1}^{\infty} \cos(\pi n) \frac{1 + 7n^2}{3n + 14n^3}$

(n) $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$

(o) $\sum_{n=1}^{\infty} \cos(\pi n) \frac{1}{n \ln(n)}$

(p) $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^3}$

(q) $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n}$

(r) $\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n}$

(s) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(t) $\sum_{n=1}^{\infty} \frac{(}{\sin} n\pi)n$

(u) $.9 - .99 + .999 - .9999 + .99999 - .999999 + \cdots$

(v) $\frac{1}{2^2} + \frac{2}{3^2} + \frac{3}{4^2} + \cdots$

(w) $\sum_{n=1}^{\infty} \left[\sin\left(\frac{n+1}{n}\right) - \sin\left(\frac{n+2}{n+1}\right)\right]$

(x) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)! - n!}{(n+2)! - (n+1)!}$

(y) $\sum_{n=1}^{\infty} \left(1 - \frac{2}{n}\right)^{n^2}$

7. Find the values of the following, or say that they are undefined:

(a) $\sum_{n=2}^{\infty} 5^{-n}$

(b) $\sum_{n=1}^{\infty}(2^n + 2^{-n})$

(c) $\int \frac{1}{x^4 + x^3}\,dx$

(d) $\int_2^{\infty} x^{-3} e^{1/x}\,dx$

(e) $\lim_{n\to\infty}(0.999)^n$

(f) $\int_2^{\infty} \frac{1}{x^2 - 6x + 9}\,dx$

8. Give an example of a series which is convergent but not absolutely convergent.

9. Give an example of two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $a_n \geq b_n$ for all n, and $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} b_n$ diverges.

10. For which values of p is the series $\sum_{n=1}^{\infty} \sin(1/n) n^p$ convergent?

11. True or false: If the sequence $\{a_n\}$ converges to L, then a_{100} is closer to L than a_{99} is.

12. True or false: If $a_n \to 0$ as $n \to \infty$, then the series $\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent.

13. True or false: If $\sum_{n=1}^{\infty} a_n$ is convergent and $a_n > 0$ for all n, then $\sum_{n=0}^{\infty} \frac{1}{\sqrt{a_n}}$ is divergent.

14. True or false: If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}(-1)^{n-1} a_n$ also converges.

15. True or False: If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ diverges, then $\{a_n + b_n\}$ diverges.

16. True or False: If the sequence $\{a_n\}$ converges and the sequence $\{b_n\}$ diverges, then $\{a_n b_n\}$ diverges.

17. It is known that the arc length of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ is $\frac{\sqrt{5}}{2} + \frac{\ln(\sqrt{5}+2)}{4}$. Does this mean that the arc length of the parabola $y = 4x^2$ from $(0,0)$ to $(1,4)$ is then $2\sqrt{5} + \ln(\sqrt{5}+2)$.

18. Who is Bill Walsh?

19. For which values of the real number q is the improper integral

$$\int_1^{\infty} x^q \sin^2(1/x)\,dx$$

convergent?

20. For which integer values of the exponent k does the sequence

$$a_n = \frac{2n^k + 3}{3n^k + 2}$$

converge? Find the limit when it exists. (The answer may depend on k.)

21. Find $\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$. (Hint: expand the terms using partial fractions.)

22. What is the recurring decimal $0.12121212\ldots$ as a fraction?

23. Set up the integral to find the area of the surface obtained by rotating the curve $y = e^x$ about the line $x = 2$ for y between $1/e$ and e, but don't evaluate.

24. Find $\displaystyle\int_0^1 x\sqrt{1-x^4}\,dx$.

25. Find $\displaystyle\int_1^\infty \frac{1}{x^{\sqrt{2}}}\,dx$.

26. Find $\displaystyle\int x^3 e^{x^2}\,dx$.

27. Find $\displaystyle\int x^2 \ln(1+x)\,dx$.

28. Find $\displaystyle\int e^x \cos^2(e^x - 1)\,dx$.

29. Find $\displaystyle\int \frac{\cos x}{\sin^2 x + 3\sin x + 2}\,dx$.

30. Find $\displaystyle\int_0^1 \frac{\sqrt[5]{x^3} + \sqrt[6]{x}}{\sqrt{x}}\,dx$.

31. Define an alternating series and state the conditions under which it converges.

32. State the error estimate for the alternating series.

33. Find $\displaystyle\int \frac{4x^2 + 9x + 3}{x^3 + 2x^2 + x}\,dx$.

34. Find $\displaystyle\sum_{n=3}^{\infty} \left(3\left(\frac{3}{4}\right)^n - 4\left(-\frac{1}{2}\right)^{n+1}\right)$