

Math 1B - Fall 2006
10/09/2006

Integrals

Ex 1. $\int \sin^2 x \cos^2 x \, dx$ - Use $2 \sin x \cos x = \sin 2x$, then use $\frac{1}{2}(1 - \cos 4x) = \sin^2 2x$.

Ex 2. $\int_0^\infty \frac{4 \, dx}{4x^2 + 1}$ - Try $x = \frac{1}{2} \tan \theta$, $dx = \frac{1}{2} \sec^2 \theta d\theta$.

Ex 3. $\int \frac{dx}{1 + \cos x}$ - Use Weierstrass substitutions, let $t = \tan \frac{x}{2}$, $dt = \frac{1}{2} \sec^2(\frac{x}{2}) dx = \frac{1}{2}(1 + t^2) dx$.

Ex 4. $\int \frac{4 \, dx}{4 + e^{2x}}$ - Let $u = 4 + e^{2x}$. Then do partial fraction.

Ex 5. $\int_0^1 \cos^{-1} x \, dx$ - Let $u = \cos^{-1} x$. $\cos u = x$.

Ex 6. $\int e^{\sqrt{x}} \, dx$ - Let $u = \sqrt{x}$. $u^2 = x$.

Ex 7. $\int_{-\pi/3}^0 \sin^2 x \, dx + \int_0^{\pi/3} \cos^2 x \, dx$ - Use half angle formula, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

Ex 8. $\int_0^{\pi/2} \sin 2x \sin x \, dx$ - Use formula, $\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$, or integrate by parts.

Ex 9. $\int \frac{x^2 + 2}{x^2 - x} \, dx$ - Use long division to reduce the order of the numerator, then do partial fraction.

Ex 10. $\int_0^4 x^2 \sqrt{16 - x^2} \, dx$ - Let $x = 4 \sin \theta$ or $x = 4 \cos \theta$.

Ex 11. $\int \frac{x \, dx}{x^2 - x + 6}$ - Completing the square for the denominator, then use trig-substitution.

Ex 12. $\int_0^\infty \frac{dx}{x^2 - 5}$ - $x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$, do partial fraction, but beware of the bad point $x = \sqrt{5}$!!

Ex 13. $\int_0^4 \frac{\ln x}{\sqrt{x}} \, dx$ - Let $u = \sqrt{x}$. $u^2 = x$, $2u \, du = dx$.

Ex 14. $\int_0^\infty \frac{\arctan x \, dx}{(1 + x^2)\sqrt{(\pi/2)^2 - (\arctan x)^2}}$ - Let $u = \arctan x$. $du = \frac{1}{1 + x^2} dx$.

Ex 15. $\int \frac{5x^2 - 5x + 5}{x^3 - 2x^2 + x - 2} \, dx$ - Denominator is $(x - 2)(x^2 + 1)$. Do partial fraction.

Ex 16. $\int \frac{dx}{\ln x}$ - Try to integrate by parts. Let $u = x$, $dv = \frac{1}{x \ln x} dx$, $du = dx$, $v = \frac{(\ln x)^2}{2}$.

Ex 17. $\int \frac{\sin^2 x}{\cos^4 x} \, dx$ - Rewrite it as $\int \tan^2 x \sec^2 x \, dx$. Then let $u = \tan x$.

- Ex 18. $\int \sin \sqrt{x} dx$ - Let $u = \sqrt{x}$.
- Ex 19. $\int e^x \ln x dx$ - Try by parts. This will get to an integral similar to Ex.16.
- Ex 20. $\int \sqrt[3]{x^2 + 1} dx$ - Let me know if you know how to do this one.
- Ex 21. $\int x\sqrt{x^2 + 4x + 13} dx$ - Completing the square and do trig-substitution.
- Ex 22. $\int (x + 1)e^x dx$ - Split the integral then integrate $\int xe^x dx$ by parts.
- Ex 23. $\int \sin^3 x \cos^3 x dx$ - Use half angle formula, like Ex.1 but a few more times.
- Ex 24. $\int_{-1}^1 \frac{(x + 2)^2}{x^2 + 1} dx$ - Do long division first, then do trig-substitution with $x = \tan \theta$.
- Ex 25. $\int \frac{1}{e^x(e^{2x} + 1)} dx$ - Let $u = e^x$. Then do trig-substitution.
- Ex 26. $\int \frac{dx}{4x^{2/3} - 4x^{1/3} - 3}$ - Let $u = x^{1/3}$. Then do partial fraction.
- Ex 27. $\int (6x - x^2)^{-1/2} dx$ - Completing square. Then do trig-substitution.
- Ex 28. $\int_0^{e^{-2}} \frac{dt}{t(\ln t)^2}$ Let $u = \ln t$. Remember to change the range values also.
- Ex 29. $\int \frac{dx}{(x + 3)(x - 2)}$ - Do partial fraction.
- Ex 30. $\int_{-1}^1 \frac{dx}{|x|^{5/6}}$ - Split at 0 but beware that it is an improper integral!!
- Ex 31. $\int \frac{dx}{x^{1/2} + x^{3/2}}$ - Let $u = x^{1/2}$. Then do partial fraction.
- Ex 32. $\int_0^\infty x^2 e^{-x} dx$ - Integrate by parts.
- Ex 33. $\int \frac{x^3 + 2x^2 - 3x - 5}{(x^2 + 3x + 2)^2} dx$ - Do partial fraction.
- Ex 34. $\int_2^4 \frac{\sqrt{16 - x^2}}{x^2} dx$ - Let $x = 4 \sin \theta$.
- Ex 35. $\int \frac{\ln x^2}{x^2} dx$ - Integrate by parts. Let $u = \ln x^2, dv = \frac{1}{x^2} dx$
- Ex 36. $\int_0^{1/2} \frac{8 - 16x}{8x^2 - 4x + 1} dx$ - Completing the square in the denominator, then do trig-substitution.