“SELECTA” BOOKS SCRAPBOOK
FOREWORDS & EXCERPTS FROM REVIEWS

Benoit B. Mandelbrot

July 1, 2004

SE ***Fractals and Scaling in Finance: Discontinuity, Concentration, Risk.***
New York: Springer, 1997, x+551pp.

SFE ***Fractales, hasard et finance (1959 - 1997)***.
Paris: Flammarion (Collection *Champs*), 1997, 246pp.

SN ***Multifractals and 1/f Noise : Wild Self-affinity in Physics.***
New York: Springer. 1999, viii + 442 pp.

SH ***Gaussian Self-Affinity and Fractals: Globality, the Earth, 1/f, and R/S.***
New York: Springer. 2002, ix + 654 pp.

SC ***Fractals and Chaos: The Mandelbrot Set and Beyond.***
New York: Springer. 2004, xii + 308 pp.

FOREWORDS

"FRACTALS AND SCALING IN FINANCE" ◇ Ralph E. Gomory (President, Sloan Foundation)

In 1959-61, while the huge Saarinen-designed research laboratory at Yorktown Heights was being built, much of IBM's Research was housed nearby. My group occupied one of the many little houses on the Lamb Estate complex which had been a sanatorium housing wealthy alcoholics.

Even in a Lamb Estate populated exclusively with bright research-oriented people, Benoit always stood out. His thinking was always fresh, and I enjoyed talking with him about any subject, whether technical, political, or historical. He introduced me to the idea that distributions having infinite second moments could be more than a mathematical curiosity an a source of counter-examples.

This was a foretaste of the line of thought that eventually led to fractals and to the notion that major pieces of the physical world could be, and in fact could only be, modeled by distributions and sets that had fractional dimensions. Usually these distributions and sets were known to mathematicians, as they were known to me, as curiosities and counter-intuitive examples used to show graduate students the need for rigor in their proofs.

I can remember hearing Benoit assert that day-to-day changes of stock prices have an infinite second moment. The consequence was that most of the total price change over a long period was concentrated in a few hectic days of trading and it was there that fortunes were made and lost. He also asserted that the historical data on stock prices supported this view, that as you took longer and longer historical data, the actual second moments did not converge to any finite number.

His thinking about floods was similar.

Benoit's ideas impressed me enormously, but it was hard to get this work recognized. Benoit was an outsider to the substantive fields that his models applied to, for example economics and hydrology, and he received little support from mathematicians who saw only that he was using known techniques. Benoit's contribution was to show that these obscure concepts lie at the roots of a huge range of real world phenomena.

Lack of recognition, however, never daunted Benoit. He stuck to his ideas and worked steadily to develop them and to broaden their range of applicability, showing that one phenomenon after another could be explained by his work. I was very pleased when I was able to get him named an IBM Fellow, and later was successful in nominating him for the Barnard Medal. After that the floodgates of recognition started to open and Benoit today is one of the most visible of scientific figures.

Surely he has earned that visibility, both for his world-changing work, and for his courage and absolute steadfastness.

"FRACTALS AND CHAOS" ◇ Peter W. Jones (Professor of Mathematics, Yale University)

It is only twenty-three years since Benoit Mandelbrot published his famous picture of what is now called the Mandelbrot set. The graphics available at that time seem primitive today, and Mandelbrot's working drafts were even harder to interpret. But how that picture has changed our views of the mathematical and physical universe! Fractals, a term coined by Mandelbrot, are now so ubiquitous in the scientific consciousness that it is difficult to remember the psychological shock of their arrival. A twenty-first-century researcher does not think twice about using a computer simulation to begin the investigation of a problem; indeed, it is now routine to use a desktop computer to search for new phenomena or seek hints about research problems. In 1980 this was very far from the case.

When a paradigm shift hits, it is rarely the old guard who ushers it in. New methods are required, and accepted orthodoxy is often turned on its head.

Thirty years ago, despite the appearance of an avant garde, there was a general feeling in the mathematics community that one should distrust pictures and any information they might carry. Computer experiments had already appeared in the undergraduate physics curriculum, but were almost nonexistent in mathematics. Perhaps this was due in part to the relatively weak computers then available, but there were other aspects of this attitude. Abstraction and generality were seen by many mathematicians as the guiding principles. There were cracks in *this intellectual foundation, and the next twenty years were to see many of these prejudices disappear.*

In my own field of analysis there had been overblown expectations in the 1950s and 1960s that abstract methods could be developed to solve a large range of very concrete problems. The correct axioms and clever theorems for abstract Banach spaces or algebras would conquer the day. By the late 1960s, groups in France and Sweden, along with the Chicago school in the U.S., had developed entirely new methods of a very concrete nature to solve old conjectures and open new frontiers. The hope of abstract salvation, at least in its most extreme forms, was revealed as naive. Especially for problems of a statistical nature, hard tools needed to be developed. (One should note that in other areas of mathematics, abstract methods have had spectacular success in solving even very concrete problems. What this means for the future of those fields is now a topic of broad speculation.)

How fascinating it is to look back on this period and observe Benoit Mandelbrot. He was looking at pictures, drawing conclusions in many fields, and being largely ignored by all. He was outside every orthodoxy imaginable.

To understand Mandelbrot's contributions to science, one must first give up the tendency to find a disciplinary pigeonhole for every scientist. What should one call someone who works simultaneously in mathematics, physics, economics, hydrology, geology, linguistics... ? And what should one think of someone whose method of entry into a field was often to find puzzling patterns, pictures, and statistics. The former could not be a scientist, and the latter could not be science! But Benoit Mandelbrot was really doing something very simple, at least at the entry point to a problem: He was looking at the pictures and letting them tell their own story.

In the mid 1500s, Galileo peered through telescopes to find astonishing celestial features imperceptible to the human eye. In very much the same spirit, Mandelbrot used the most modern computers available to investigate phenomena not well studied by closed formulas, and out popped strange and unexpected pictures. Furthermore, he worked with the idea that a feature observed in a mathematics problem might be related to "outliers" in financial data or the observed physics of some system. Perhaps these rare events or outliers were not actually so rare at all; perhaps they were even the main feature of the system!

After getting his foot in the mathematical door, Mandelbrot would start the next phase of research, erecting a mathematical framework and doing the hard estimates. Try today to explain to the scientifically literate high-school student that the beautiful fractal pictures on a computer screen are not interesting, at least not to be trusted, and try asserting that the fractals arising in wholly different problems are similar due just to chance.

While the aversion to looking at pictures has faded, there is still confusion as to why Mandelbrot's early works on fractals, e.g., his book The Fractal Geometry of Nature, generated such wild popularity in the general scientific community. One does not see on every page the "theorem-proof" methodology of a mathematics textbook. Furthermore, though one can easily find theorems and rigorous proofs in the book, the phenomena and pictures discussed may seem to a mathematician to be unrelated, because there is not necessarily an exact theorem to link any two of them.

What a poor world we would live in if this were the only permitted method to study the universe! Consider the plight facing a working biologist, where all data sets are dirty and causality difficult to determine. Should one demand a theorem in this situation? Should a geologist looking at rock strata search first for a theorem, when the formalism of multifractal measures might be more important? An old tradition in science is to seek first a description of the system at hand; this apparently simpler problem is usually much more difficult than is generally believed. Few doubt that Kepler's laws would have been formulated without his first seeking patterns by poring over reams of data.

Perhaps, however, the pictures studied by Mandelbrot arose randomly, and any connection to interesting science is just a coincidence. The Mandelbrot set M offers an instructive example. Despite twenty years of intensive research by the world's best analysts, we still do not know whether M is locally connected (the MLC conjecture), and progress on this problem has rather ground to a halt. This is now seen as one of the most central problems of complex dynamics, and the solution would have many deep consequences. The geometry of M is known *to be devilishly complicated; M. Shishikura proved that the boundary has dimension equal to two.*

We know today that the "Sullivan dictionary" provides many analogues between iteration of rational functions and the theory of Kleinian groups, but there is very much that remains open. For example, we do not know whether it is possible for either a Julia set or a limit set (of a Kleinian group) to have positive area unless it is thefull sphere. If all Julia sets from quadratic polynomials have zero area, then the Fatou conjecture on density of hyperbolic systems would be proven for quadratics. It is also known that MLC implies both the Fatou conjecture for quadratics and the nonexistence of certain (but not all) Julia sets of positive area.

Another example is furnished by the Brownian boundary that is the subject of Plate 243 of The Fractal Geometry of Nature. Arguing by analogy and examination of simulations, Mandelbrot proposed that the Brownian boundary has dimension 4/3 and serves as a model for (continuous) self-avoiding random walks (SARW). The 4/3 conjecture was only recently solved by the spectacular work of G. Lawler, O. Schramm, and W. Werner. Their proof relied heavily on the New processes called SLE that Schramm invented. We now know that SLE (8/3) represents the Brownian boundary. This also proves another prediction of Mandelbrot that the two sides of the Brownian boundary are "statistically similar and independent." One of the major challenges in probability theory is to prove that SARW exists, and the new conjecture is that it can be identified with SLE (8/3).

The study of multifractals is another area where Mandelbrot played a leading role. Through multiplicative measures with singular support were known in certain areas of Fourier analysis and conformal mappings, their fine structure had not been examined, and they were virtually absent in discussions of physical problems until the work of Mandelbrot. He was also the first to write down f (alpha) in the form of normalized logarithms of large deviation probabilities.

The status of these problems may be open, but the beautiful pictures, now easily reproduced by the aforementioned high-school student, continue to fascinate and amaze. What we see in this book is a glimpse of how Mandelbrot helped change our way of looking at the world. It is not just a book about a particular class of problems; it also contains a view on how to approach the mathematical and physical universe. This view is certain not to fade, but to be part of the working philosophy of the next mathematical revolution, wherever it may take us.

REVIEWS

Acta Scientiarum Mathematicarum ◇ Szeged, Hungary ◇ SE ◇◇ László I. Szabó

…A mixture of newly-written material, old articles and contributions from other authors. The text is centered around 3 successive models of price variation….

While some of the author's views and hypotheses are debated by some economists, Mandelbrot's original insights have unquestionably contributed in a substantial way to our understanding of the economic world. This book is a timely work in the age of econophysics.

American Mathematical Monthly ◇ SE ◇ April, 1998 ◇ KB

Highlights a new classification of forms of randomness into "states" that range from mild to wild; a useful classification of prices' departures from Brownian motion, into Noah and Joseph effects and their combination; a broad panorama of old and new forms of self-affine variability. Theme: although prices vary wildly, scaling rules hold ensuring financial charts are examples of fractal shapes.

AFP: L'Information à la Source ◇ SEF

Peu de personnes savent que la géométrie fractale est née des travaux que Benoît Mandelbrot, son "inventeur", avait consacré à la finance dans les années soixante. Près de quarante ans plus tard, le mathématicien convie … le lecteur à utiliser ces mêmes concepts d'imprévisibilité pour décrypter le comportement des cours de la Bourse et tenter une évaluation réaliste des risques financiers. Tout avait commencé pour Mandelbrot, en 1960, [quand] il découvrit une symétrie entre les grandes et les petites échelles: la courbe d'évolution sur une semaine était en effet semblable à celle d'une dizaine d'années. Ce sera le début de l'épopée de ces objets rugueux, poreux ou fragmentés et invariants à toutes les échelles, nommés "objets fractals". Cet insatiable curieux a depuis appliqué sa vision à [quantité d'autres domaines].

American Mathematical Monthly ◇ SE ◇ 1998 ◇ KB

…Theme: although prices vary wildly, scaling rules hold ensuring financial charts are examples of fractal shapes.

Capital ◇ SEF ◇ Janvier 1998 ◇ Jean-François Rouge

LA MÉTHODE D'UN MATHEUX CÉLÈBRE POUR GAGNER EN BOURSE. *Le professeur Ian Malcolm, le héros de "Jurassic Park" et du "Monde perdu" … emprunte la plupart de ses idées à un mathématicien français de 73 ans, Benoît Mandelbrot… Malcolm invoque en effet une théorie mathématique, dite du chaos... On parle aussi de … "géométrie fractale". Le père de cette science en vogue (et donc souvent trahie) vit à New York et enseigne à l'université de Yale.*

Jamais, depuis Einstein, un théoricien dont les écrits sont pourtant de l'hébreu pour la plupart des mortels, n'avait bénéficié d'un tel effet de mode, y compris en librairie…

[Dans son nouveau livre], Mandelbrot renoue avec un thème de recherche qu'il avait abordé à ses débuts, dans les années 60: la prévision et, surtout, la prévention des chocs boursiers. Les fractales, qui permettent par ailleurs de mesurer précisément la longueur d'une côte accidentée ou de prévoir la croissance de la végétation dans une forêt, peuvent en effet déterminer la volatilité d'une action ou d'un indice. Une connaissance qui permet ensuite de fixer les réserves nécessaires à une banque ou à une compagnie d'assurances pour s'en tirer au mieux en cas de tempête boursière ou monétaire.

Complexity

Computers and Mathematics with Applications ◇ SE ◇ **35**, 1998(5).

EMS-European Mathematical Society Newsletter ◇ SE ◇ Vol.34, p.40, 1999 jh ◇ K.

Presents an alternative approach to the analysis of financial data, or more generally, to any data set possessing features like financial time series. Is recommended to any mathematician and/or financial analyst who wishes to learn more about the variety of alternative models and to avoid using just the classical methods.

L'Enseignement mathématique ◊ SN ◊ 45 (3/4), 1999.

Le Figaro ◊ SFE ◊ Samedi 13 décembre 1997 ◊ Georges Suffert

LE DETERMINISME INVISIBLE DES COURS DE LA BOURSE.

LES SURPRENANTES INTUITIONS DE BENOIT MANDELBROT: L'inventeur des fractales s'attaque aux lois de la finance. Ou pourquoi l'ordre caché du hasard renvoie dos à dos pessimistes et optimistes.

Benoît Mandelbrot est un personnage qui échappe à toute classification. A première vue, on pourrait le prendre pour l'héritier conjoint d'Einstein et du professeur Nimbus. Il donne l'impression de se mouvoir dans un univers sans grand rapport avec le nôtre; il voit, il pressent, il formule d'étranges rapports mobiles entre des formes apparemment figées à jamais. Depuis des années, il détecte le désordre invisible qui se dissimule à la surface des objets, dans des structures de lignes apparemment droites. Il est l'un des fils de l'aléatoire, il poursuit des ruptures que nous ne percevons même pas. Et d'un même mouvement, il reconstitue une forme de régularité, d'apparent déterminisme derrière le fourmillement des apparences désordonnées. Les fractales furent la découverte qui obligea ses collègues mathématiciens, géologues et physiciens à admettre que Mandelbrot avait de surprenantes intuitions. On se souvient de l'image désormais classique qui popularisa ces fameuses fractales: il suffit d'observer sur une carte la côte de la Normandie… Aujourd'hui, Benoît Mandelbrot, polytechnicien et normalien, est reconnu, aux Etats-Unis du moins, comme un exceptionnel observateur du réel. Il commence par voir, puis il fait intervenir les mathématiques. Si l'on veut mesurer l'importance de son apport, il suffit d'ouvrir les ouvrages actuels sur le chaos ou les mélanges (chez Odile Jacob): le système des fractales sous-tend l'essentiel des descriptions présentées.

Mais aujourd'hui Mandelbrot se lance dans une chasse particulièrement fascinante: les cours de Bourse obéissent-ils à des lois connues? …

L'aléatoire ordinaire*: tout se passe comme s'il y avait du "désordre" dans les variations des Bourses… ce problème fascine, semble-t-il, Mandelbrot. Il a commencé par s'interroger sur le modèle proposé par Louis Bachelier en 1900. Les prix, d'après ce chercheur lointain, se promènent dans l'aléatoire ordinaire. D'où un mouvement brownien classique. En 1963, puis en 1965, Mandelbrot propose ce qu'il étiquette sous le terme de "brownien fractionnaire".*

Hasard sauvage*: à cette date, notre chercheur a repéré sur courbes et équations deux types distincts de hasard sauvage: les événements catastrophiques isolés (effet Noé) et les alternances régulières de vaches grasses ou maigres (effet Joseph). On aboutit à des courbes imprévues, très différentes de celles de Bachelier et de ses successeurs…*

On fera remarquer que pour le moment, la théorie de Mandelbrot ne sert à rien: pas question de gagner en Bourse sans risque d'erreurs. Mais l'intérêt réel n'est pas là: ce que notre chercheur commence à distinguer, ce sont ces lois invisibles et inconnues qui gouvernent ce que nous appelons hasard; encore une fois, une espèce de déterminisme invisible qui surgirait de l'aléatoire lui-même. Il y aurait un ordre étrange dans l'immense désordre de l'univers apparent.

Un petit ouvrage à lire pour ceux qui ne reculent pas devant les courbes et quelques équations. Mais on peut sauter ces passages.

The Guardian ◊ SE ◊ 11.12.97 ◊ Clive Davidson

MANDELBROT'S ROLLERCOASTER.

THE DISCOVERER OF THE CHAOS THEORY HAS PUBLISHED HIS IDEAS ON WHY STOCK MARKETS CRASH. SCALING CAN PRODUCE WILDLY RANDOM MOVEMENTS

…Everyone knows that every so often the markets experience swings of mood, when prices jump in a flurry of trading. But classical market theory says they shouldn't. Prices should make small random movements, rather like particles in a solution bombarded by surrounding molecules. If the markets always followed such "Brownian movement"…--prices would steadily zigzag their way up or down in response to changing economic conditions.

But according to Benoit Mandelbrot, the mathematician who discovered fractals, we don't have to abandon the notion of randomness to create a model of the markets that more *accurately reflects their reality. In his new book,…he argues that fractal-based models give a more realistic picture of financial risk.*

One of the principles of fractals is that apparently simple processes can generate unexpectedly complicated and structured forms. We see these in nature, from the shape of plants to geological formations.

Mandelbrot demonstrated the process in the seventies, when he used a computer to produce striking and complex images from relatively simple equations that became known as the Mandelbrot Set.

Although Mandelbrot is best known for these images and for his work on fractals in the physical world, he first stumbled on the phenomenon in the financial markets.

In the fifties, he set out to investigate a piece of financial folklore that suggested that if you took away the time scales of a series of graphs of prices plotted over different intervals you could not tell which was which.

This self-similarity at different scales is a feature of fractal systems. Mandelbrot went on to show that one of the features of such scaling systems is…that the changes in such systems are not evenly spread over time but tend to happen in concentrated bursts.…A system based on the principles of scaling can produce what Mandelbrot calls "wildly random" movements, just like the price crash and recovery on October 23.

Financial professionals are most comfortable when market conditions are mild, with small fluctuations in prices. They tend to supplement their Brownian movement models with "stress tests," in which they look at what would happen to their portfolios if there was a rerun of the crash of 1987 and so on. Mandelbrot's ideas offer a way to build market models that include periods of calm as well as price hikes, crashes and the like.

…One of the problems for Mandelbrot in the sixties was the lack of market data, computers and statistical tools. So it was difficult to test his assertions and they were largely ignored.

But two decades later, things had changed….

Mandelbrot's theories no longer seem so wild to the financial industry.

IBM Research ◇ SE ◇ Number 3, 1997

After a long detour through the rest of the universe, Benoit Mandelbrot's exploration of fractal phenomena has come back to its roots. While it was not until 1975 that he coined the term "fractal" – to refer to mathematical and natural objects characterized by the same extreme degree of irregularity at all scales – the underlying ideas had been germinating for much longer. His just published book itself consists of old and new material, in roughly equal parts, including his reprints of articles on finance and economics from 1960 to 1973. In a long, multichapter introduction, Mandelbrot places the evolution of his work in context and explains the new picture of economic phenomena that his ideas entail…

As in his other excursions into fields as diverse as condensed-matter physics, mathematics, linguistics, geophysics, fluid dynamics and astronomy – to name a few – Mandelbrot brought to the study of finance and economics a gift for geometric insight and a capacity to seize on and synthesize ideas that others had either overlooked or failed to see could be applied in a novel context.

At the heart of Mandelbrot's approach to economics is a contrast he draws between different "states of randomness." From his viewpoint, the randomness dealt with in traditional physics and used by Bachelier in his Brownian-motion, or random-walk, model of price variations is mild, whereas financial reality is characterized by the state of "wild randomness." Thus, he argues, there is no underlying equilibrium whose fluctuations average out; rather, price changes experience cycles of turbulent behavior.

Yet, underlying this extreme randomness are invariance principles arising from a generating process that remains constant in time. The result is that a graph of price changes is invariant in a statistical sense under displacements along the time axis and under change of scale. Such scaling, or self-affinity (a notion close to self-similarity), is, of course, the tell-tale sign of a fractal.

Mandelbrot's goal in creating economic models was to obtain some degree of understanding of phenomena that seemed impervious to mathematical description. By showing that the wild randomness of the data can be modeled more accurately than previously believed in a way that *does not depend on a variety of ad hoc "fixes," Mandelbrot has also produced practical tools to evaluate the inherent risks of financial trading. The search for understanding must continue, says Mandelbrot, "but financial engineering cannot wait for full explanation." Meanwhile, the increased breadth, depth, and accessibility of Mandelbrot's ideas will undoubtedly spur new efforts in a field that affects us all.*

International Statistical Institute Short Book Reviews ◇ SE ◇ P.A.L. Embrechts (ETH, Zürich, Schweitz)

ITW Nieuws (Niederlande) ◇ SE ◇ Jaan van Oosten

Jahresbericht der Deutscher Mathematiker Vereinigung ◇◇SE ◇◇Band 101(2)
M. Schweizer (Technische U., Berlin, Deutschland)
As a partial collection of selected papers, the book has certainly quite some historical value and interest. It also presents a remarkable panorama of ideas and clearly shows what Paul Samuelson called Mandelbrot's "incorrigibly original mind." In other aspects, however, I found some deficiencies. Most important among these is probably the lack of a balanced presentation. While some of the personal comments are entertaining, one misses at least an overview of other work that has been done in the area of Mandelbrot's research in finance. There is hardly any mention of the evolution between the sixties and today, and there is no effort to provide a perspective of the field as a whole. As a consequence, some comparisons are quite clearly biased and also not up to date…

In summary, this book is a useful collection of Mandelbrot's most important papers on modeling in finance, supplemented by some more recent ideas on the role of scaling rules in that field.

Journal of Economic Literature and e-JEL, JEL on CD, and EconLit (Volume 38, no.3)
Elaborates on the tendency of stock market price changes to concentrate in turbulent periods in a series of newly written essays followed by reprinted papers that give historical depth and add technical detail. Discusses discontinuity and scaling, their scope and likely limitations; sources of inspiration and the historical background; states of randomness and concentration in the short, medium, and long run; self-similarity and panorama of self-affinity; proportional growth and other explanations of scaling; and a case against the lognormal distribution. Three sections of reprinted papers examine personal incomes and firm sizes; test or comment on the author's 1963 model of price variation; and present steps beyond the 1963 model.

Journal of Economic Literature ◇ SE ◇ (Volume XXXIX) June 2001
Philip E. Mirowski (University of Notre Dame)
Benoit Mandelbrot is an imaginative scholar, but one whom those equipped with firm disciplinary loyalties have found it a struggle to understand. This has been a problem across the disciplinary spectrum, although in economics it has assumed one of two forms: there are the neoclassical finance theorists, who argue against what they have perceived as his notions precisely because they clash with received microeconomic theory (although empirical controversies have also played a role); and then there are the self-designated "econophysicists," refugees from the natural sciences with no particular doctrinal orthodoxies to defend, who have been attracted to his work precisely because of its undeniable influence in the physics of turbulence, diffusion processes, semiconductors, and elsewhere. It used to be that Mandelbrot would confound both his supporters and detractors by insisting upon a third stance, which went roughly: research should be guided by the precept that the geometric character of any given phenomenon should be the primary heuristic, combined with a fearless acceptance of indeterminism; no inquiry should be prematurely stifled by either entrenched dogmas or by ill-conceived physics envy. In economics, this amounted to repeated exercises arguing that empirical price distributions were fat-tailed, exhibiting long dependence, and altogether more ragged than allowed in conventional econometric models.

This volume, retailed as a collection of previously published papers over the past four *decades but actually more like a running commentary with selective revisions and new additions, suggests that Mandelbrot himself has moved closer to the econophysicists, perhaps due to his own success in convincing the physicists and relative failure in connecting with economists. Because of this shift, I doubt that any financial economist picking up this book would readily grasp the tenor of Mandelbrot's recent thought without a prior introduction to a primer on multifractals, perhaps augmented with his more recent Selecta volume (1999, Multifractals and 1/f Noise, NY: Springer Verlag).*

If there has been a common thread throughout Mandelbrot's economics, it is the conviction that "the essential role of a Bourse is to manage the discontinuity that is natural in financial markets" (ibid, p. 68). His attempt to express this geometric insight has assumed two formats over time, both linked but not well integrated with each other, undergoing shifts in emphasis throughout the period represented in this volume. In the first, one approaches time series of prices as an unabashedly stochastic phenomenon and asks for the most cogent and parsimonious interpretation of the evidence. In 1963, Mandelbrot caused a furor by asserting drat the Gaussian model was a poor fit, and that the more general Levy-stable family of distributions, derived from a more general limit theorem, gave a better characterization. Over time, Mandelbrot has backed away from this claim as it has come under sustained fire from within the economics profession, but also as he came to appreciate that various stochastic characterizations constituted a continuum, with the Gaussian at one extreme, the lognormal an intermediate case, and Levystable distributions as the "wild" other extreme. Given the family resemblances, it was deemed unlikely that the question of stochastic characterization could be presented as a dichotomous 'either/or,' much less distinguish between long dependence and a marginal distribution with infinite variance, and therefore Mandelbrot now has relinquished many of his earlier claims for generality and simplicity. For instance, he no longer champions a fearless indeterminism (p. 16), and indeed, has forsworn the goal of a general stochastic characterization applicable to all markets (p. 13).

In the second format, the geometric characterization of the price series assumes pride of place while probability takes a backseat; and Mandelbrot reminds us that it was his early work on finance that led to his more famous work on fractals, rather than vice versa. Yet sometime in mid-career, Mandelbrot realized that price time series were not strictly self-similar, but rather self-affine (literally, globally non-fractal). This led to his more recent theoretical commitment to stationarity and scaling as the effective equivalents of conservation laws in physics: unshakable theoretical commitments, whether they areempirically true or not, or as he writes, "Hamiltonians allow physics to explain scaling. But those laws have no counterpart in finance" (p. 113). Some will feel we are left with a radically undermotivated modeling strategy, which consists of producing computer simulations of price time series which mimic the movements of observed prices by means of deterministic iterative algorithms which squeeze, slice, dice and otherwise massage simple splines and, more curiously, the time axis as well. This constitutes the "model" that Mandelbrot apparently now favors, at least for first differences of corporate share prices, a combination of fractional Brownian motion in multifractal time. Hence we are left with the portrait of someone who rejects the orthodoxy in modern finance because he believes his geometric characterization contradicts lognormality, ARCH models, the Ito calculus, and most of the rest of the accoutrements of financial economics. By all accounts, he no longer grapples with actual empirical price series as he did in the 1960s; in this second phase we are squarely confined to the realm of stylized facts.

In the shift from the first to the second narrative, we seem to have come quite a distance from the Mandelbrot of the 1960s, even though he himself gives little indication of any analytical rupture. The absence of specific model motivation was acceptable in the 1960s, since the point of the crusade then was to insist that the unwavering adherence to the Gaussian distribution was neither so innocent nor harmless as economists (still) appear to think. Mandelbrot's retreat from the Levy-stable generalization in the interim, however, has left him in much the same uncomfortable position as those whom he criticizes. For instance, his recent model resembles Peter Clark's (1973, "A Subordinated Stochastic Process Model with Finite Variance," Econometrica, 41, pp. 135-55) subordinated stochastic process to a much greater degree than he might be willing to admit; and Clark's model was explicitly intended to provide a more neoclassically-friendly alternative to Mandelbrot's original assertions. The recourse to *multifractals appears to multiply parameters in much the same way that ARCH and other curve-fitting techniques do; and hence it becomes much harder to distinguish which simulation exercise should be deemed as coming off 'better' in the race to mimic price movements, a dire "Red Queen" tournament that might be considered as having begun with the chartists.*

What seems missing from the controversy is adequate consideration accorded to what renders the phenomena specifically economic. For instance, physicists can legitimately entertain the notion that time is a relativistic variable; but why should economists do so? Further, if we must assume for the purposes of statistical estimation that the Brownian motion of prices is statistically independent of the temporal driver, doesn't this conflict with the ingrained notion that price and quantity are interrelated in a specific market? And while there are a surfeit of lapsed physicists drawing their paychecks from brokerage houses, is there something more than this that specifically encourages these particular formalisms in finance, in the absence of any comparative study of the possible self-affine character of price movements in retail or consumer markets? Mandelbrot's geometric eye should provoke a more serious reconsideration of the aims and goals of quantitative empiricism, something sorely lacking since the old "Measurement without Theory" controversy.

Kwantitatiere Methoden (Niederlande) ◊ SE ◊ E. Omey

Le Légitimiste ◊ SFE ◊ Novembre 1997

La collection "Champs" chez Flammarion, très bon ensemble de textes de haute vulgarisation scientifique, vient de faire paraître deux livres qui valent la peine d'être lus, même s'ils ne sont pas, il s'en faut, des reflets de nos idées ou de nos préoccupations… [Le deuxième de ces] ouvrages est signé Mandelbrot, mathématicien connu pour ses travaux sur les fractales (objets mathématiques liés à la théorie en vogue du chaos): Fractales, Hasard et Finance. Par son caractère (relativement) appliqué aux questions économiques, ce livre est (toujours relativement) beaucoup plus abordable que bien d'autres et constitue ainsi une bonne entrée dans une théorie qui marque considérablement notre temps. La lecture nécessite cependant un bagage scientifique solide, particulièrement en probabilités, discipline liée, comme chacun sait au hasard.

Libération – Eureka ◊ SE ◊ Mardi 24 Février 1998 ◊ Sylvestre Huet

EN APPLIQUANT SA THEORIE DES FRACTALES, LES CHIFFRES QUI SERVENT A ECRIRE LE CHAOS, A LA FINANCE, BENOIT MANDELBROT AVERTIT QUE LE RISQUE BOURSIER EST PLUS GRAND QUE NE LE DISENT LES COURTIERS.

◊ Votre livre distingue les hasards "bénins et sauvages". Que signifient-ils?

En science, le hasard n'est pas le "sort" personnifié, mais uniquement une mesure de notre ignorance. Dans certains domaines comme la physique classique, cette ignorance est contrôlable par les mathématiques. C'est un hasard que j'ai baptisé bénin. Si l'on attend suffisamment longtemps, on découvre l'ordre caché. Si vous jouez à pile ou face vous aurez une statistique très simple de 50-50. Si vous écoutez une radio sans viser un émetteur vous entendez un bruit de fond. Il provient du hasard, mais on ne peut l'éliminer car il bouge sans arrêt autour d'une intensité bien fixe.

Mais il y a un autre hasard, le sauvage. Il est très vilain, car il ne permet pas de raisonner en termes de moyennes. Si vous prenez dix villes de France au hasard et si vous ratez Paris, Lyon et Marseille, vous allez faire chuter la taille moyenne dans votre échantillon. Si vous prenez dix villes dont Paris et neuf villages, la moyenne n'autorise aucune conclusion sur les populations de villes tirées au hasard. Un autre exemple spectaculaire, c'est la distribution des galaxies dans l'Univers. Classiquement, les astronomes partaient de l' idée que l'Univers, à très grande échelle, présentait une distribution uniforme des galaxies. Donc que la notion de densité moyenne de matière avait un sens. Or, plus on voit loin, et plus on distingue d'énormes vides, qui démolissent cette idée. En fait, la distribution des galaxies semble être un exemple merveilleux de hasard sauvage. Les techniques usuelles ne permettent de tirer aucune conclusion sûre. Il faut donc changer de méthode mathématique.

◊ C'est là ce que vous proposez d'utiliser les mathématiques fractales. Que sont-elles?

Une fractale est un objet mathématique que l'on peut couper en petits bouts et dont chaque bout présente la même structure que le tout. Le chou-fleur est une très jolie fractale naturelle. Chaque morceau que vous détachez présente la même structure que le tout, et ainsi de suite. Avec deux bornes de taille, supérieure et inférieure, évidemment, alors que son analogue mathématique peut être sans limites. L'homme aime bien cette vision hiérarchique, l'emboîtement des structures. Le point central de ma découverte, c'est qu'il ne s'agit pas seulement d'une astuce mathématique mais que c'est une propriété fondamentale de très nombreux objets naturels. Quand on combine hasard sauvage et fractalité, on a d'un côté une mauvaise nouvelle: difficulté accrue de la compréhension et de la prévision. Et, de l'autre, une bonne nouvelle: si l'objet, ou la dynamique, peut être décrit à l'aide d'un nombre fractal, on se retrouve avec un objet mathématique relativement simple, puisqu'il est fondé sur une invariance, le concept de base de la science. Dans une fractale, il s'agit d'invariance d'échelle: les propriétés sont les mêmes, quelle que soit l'échelle à laquelle on les regarde.

◇ Avec cela, vous pouvez aider les financiers à gérer le "hasard sauvage" de la Bourse?

Prévoir les cours de la Bourse avec les mathématiques n'est pas un espoir sérieux. Si une telle formule existait, son objet disparaîtrait d'ailleurs automatiquement, car tous les acteurs boursiers auraient la même information sur l'évolution des actions, ce qui changerait leurs décisions. Les grands changements de prix sont, pour presque tout le monde, imprévisibles. Mais, si on pouvait mieux en évaluer statistiquement les risques, on pourrait les amortir. Or, les fractales permettent justement d'étudier simplement les cours, le comportement d'une Bourse, en indiquant son degré de variabilité par un seul chiffre, la dimension fractale des courbes de prix des actions. Un chiffre est compris entre 1 (la ligne droite) et 2 (la surface), qui peut aider à mieux apprécier le risque de chute brutale. Mon travail ne promet pas de bénéfice pour un particulier, mais il peut aider à mieux chiffrer les réserves obligatoires des banques, à assurer des investissements de fonds de pension ou à réguler un peu le marché, le protéger des soubresauts et des faillites retentissantes. On veut, en somme, moyenner les risques pour avoir le moins de fluctuations possible. Mais, pour faire cela, il faut estimer les risques de manière correcte. Or, l'expérience le prouve, les risques sont beaucoup plus grands que ne le disaient les théories économiques. Regardez le nombre de faillites totales de grands portefeuilles, tenus pourtant par des experts.

◇ Pouvez-vous prévoir les bulles boursières, ces hausses des cours qui semblent déconnectées de l'économie réelle, comme celles qui viennent d'imploser en Asie?

En 1966, j'ai décrit dans un article une Bourse très rationnelle et pourtant telle que les prix paraissent monter sans arrêt, toujours rationnellement, puisque à chaque moment, la persistance de la montée est plus probable que son interruption. Mais, corrélativement, la valeur de la chute possible monte elle aussi. Et, plus on attend, plus la chute est rude. Donc le risque augmente constamment. Cela était contenu dans les formules mathématiques produisant ces fameux courbes fractales qui ressemblaient tant aux courbes réelles. Lors du krach du 19 octobre 1987, des financiers m'ont dit: "C'est exactement le comportement que tu décrivais en 1966." Anticiper la présence de grosses bulles, montrer qu'elles peuvent être parfaitement rationnelles est un triomphe de la théorie. D'autant plus, finalement, que l'on peut, avec les mathématiques fractales, les décrire et prévoir pour elles un risque plus élevé que ne le pensaient les théoriciens de la finance… sans pour autant expliquer les raisons économiques de ce que j'observe, ce qui n'est pas de mon ressort.

Mathematical Reviews ◇ SE ◇ Bing Hong Wang (PRC-HEF-MP; Hefei)

Mathematical Reviews ◇ SN ◇ Daniel J. M. Schertzer & Shaun M. Lovejoy

The book primarily emphasizes the importance of two papers: J. Berger and B. B. Mandelbrot, (1963), and B. B. Mandelbrot (1974). The former is… presented as a main historical step to understanding clustering/intermittency with the help of strongly non-Gaussian noises…. This indeed corresponds to an important step towards the use of fractal dimension to characterize intermittency.

The 1974 paper… deals with a broad and significant generalization of the pioneering multiplicative cascade model of A. M. Yaglom (1966) which was built up in order to understand turbulent intermittency. Indeed, it is physically argued that the "microcanonical" constraint (i.e.

strict conservation of the energy flux) should be replaced by a less demanding "canonical" one (i.e. conservation of the energy flux under statistical average). The latter yields a much more variable energy flux.

That paper is followed by a revised English translation of a more mathematically oriented two-part companion paper [B. Mandelbrot (1974)] that presents several conjectures. The next chapter---a "guest contribution"---corresponds to an English translation of J.-P. Kahane and J. Peyrière, Advances in Math. 22 (1976), presents exact mathematical results on multiplicative cascade processes. …

There is a sharp difference that one needs to underline between [J. Berger and B. B. Mandelbrot, op. cit.] and [B. B. Mandelbrot, op. cit.], and the corresponding parts of the book: the former deals with additive processes [Comment by BBM: Not in the least], the latter with multiplicative ones….

Mathematical Reviews ◇ SH ◇ Michèle Mastrangelo-Dehen

This book is a complete and encyclopaedic synthesis of problems and themes in self-affinity fractals, multifractal geometry and globality. In the early 1960s, Mandelbrot began to explore these subjects; the aim of this book is to contribute to the scope of knowledge in and the development of the field. It addresses numerous old and new problems arising in many varied disciplines: mathematics, physics, engineering, hydrology, climatology, statistics, economics, finance, etc. The first third of the volume consists of extensive introductory material written especially for this book. At the beginning, there is an overview of recent work in fractals and multifractals….

At the end, a very exhaustive and complete bibliography on the different connected subjects (with more than 500 references) may be very useful for researchers or users.

Monateshefte für Mathematik ◇ (Austria) ◇ SE ◇ 2000 ◇ P. Schmitt (Wien)

Today, the finance market is a major target for applications of mathematics. In this volume of his Selecta, Benoit Mandelbrot, (also) a pioneer in this field, assembles his papers on this subject (from 1960 onwards) and supplements them with newly written chapters. Thus it is both a monograph of the subject (by one of the masters), and an interesting source of its history.

Nature ◇ SE ◇ February 19, 1998, Vol 391

◇ Ian Stewart (Mathematics, U. of Warwick, Coventry, UK)

MONEY SPINNER ◇◇*In the late 1950s and early 1960s, Benoit Mandelbrot, then a young and relatively unknown researcher at IBM's T. J. Watson Research Center, devoted a lot of his time to problems in economics and finance. His ideas in these areas formed a tiny part of a huge body of work, ranging from rainfall statistics to linguistics, that led him to create the concept of the fractal, for which he is now famous all over the world.*

Fractals and Scaling in Finance *brings Mandelbrot's work full circle, applying today's mature fractal geometry to the problems that plagued him 40 years ago. It is a typically Mandelbrotian mix of reprinted papers, commentary, unpublished results and new work hot off the press. Its focus is simple: what is the structure—if any—of financial data?… Faced with [the world's stockmarkets'] unpredictability, classical mathematics took one look at the financial world, deemed it to be random, and the paradigm was set. The great triumph of modern financial mathematics—the famous Black-Scholes equation of 1972, without which there would be no derivatives market—is based on a purely stochastic model…*

The evidence that market data possess hidden structure is becoming overwhelming… But what kind of structure do market data possess? According to Mandelbrot, their central feature is scaling. Roughly speaking, the small-scale fluctuations of the market mimic the large-scale ones, but on a compressed timescale…This implies, for example, that jumps in market value can be bigger, and more rapid, than conventional statistics would allow…In a way, the entire book is the story of Mandelbrot's intellectual voyage into power-law territory, as he refined his early observations into more sophisticated, but always simple and elegant, models of the irregularities of financial time series…This is not an easy book to read, but once one gets into the flow of ideas it is rewarding and full of insight, and should be on every chartist's bookshelf.

There is one weakness, though, that will be apparent to any child of the computer age: a neglect of modern data. Mandelbrot explains that, although extensive data are now available, he is "ill equipped for empirical work." Perhaps…but isn't that what postdocs are for?

Nature ◇ SE ◇ July 1, 2004, Vol 430

◇ Kenneth Falconer (Pure mathematics, U. of St. Andrews, St. Andrews, UK)

INFINITE BEAUTY ◇◇*It was just over 20 years ago that the Mandelbrot set took the world by storm. Pictures of extraordinary complexity and beauty appeared in scientific and glossy magazines, on the walls of art galleries and classrooms, on posters and even on tablemats. With the increasing availability of personal computers, drawing the Mandelbrot set became a standard exercise for those learning programming, and it was frequently an addiction for computer buffs, who were able to explore its intricacy by forever homing in on parts of the structure. Perhaps it is not surprising that such a simple procedure enabling almost anyone to produce an object of immense sophistication and attractiveness caught the public imagination.*

The definition of the Mandelbrot set, denoted by M*, is indeed extremely simple. Given a complex number* c*, start at the origin 0 and follow the trail of points obtained by repeatedly applying the transformation* $f(z) = z^2 + c$*, that is, the sequence 0,* c*,* $c^2 + c$*,* $(c^2 + c)^2 + c$*... If these points never go far away from the origin then* c *is in* M*, but if they wander off to infinity,* c *is not in* M*. This straightforward check allows one to scan across a region of the complex plane to determine the extent of* M*.*

Crude pictures of M *show a main cardioid surrounded by circular 'buds' of decreasing size. But more detailed investigation, pioneered by Benoit Mandelbrot in 1980, reveals much, much more: the buds are all surrounded by smaller buds, which in turn support even smaller ones, and so on. Homing in on the boundary of* M *reveals a menagerie of multi-branched spirals, dragons and seahorses. Hairs of imperceptible fineness extend from the buds, holding along their lengths minute replicas of the entire Mandelbrot set.*

Is the Mandelbrot set just a pretty curiosity? Far from it. It is a fundamental parameter set that encodes an enormous amount of information about nonlinear processes. First, the position of a complex number c *relative to* M *tells us a great deal about the iteration of the quadratic mapping* $f(z) = z^2 + c$*. The (filled-in) 'Julia set' at* c *consists of those complex numbers* z *whose iterates under repeated application of* $f(z) = z^2 + c$ *never wander far from the origin. This Julia set comprises a single piece precisely when* c *lies in the Mandelbrot set. (Interestingly, this topological dichotomy was noted by Pierre Fatou and Gaston Julia in 1918–19, but it was many years before its real significance and delicacy was appreciated.)*

Much more than this, the exact position of c *in* M*, such as the bud in which it lies, gives a very full description of how the iterates of* f(z) *behave: for example, whether there are periodic cycles. Even more surprising is that, although defined in terms of the simplest of nonlinear maps,* $f(z) = z^2 + c$*, the Mandelbrot set is 'universal' in that it underlies the behaviour of very large classes of more complicated nonlinear mappings, the likes of which crop up throughout modern mathematics and its applications.*

The emergence of the Mandelbrot set in 1980 led to a flurry of activity among mathematicians trying to understand its structure and significance, resulting in some of the most impressive advances in pure mathematics in recent years. In 1982, Adrien Douady and John Hubbard proved the (far from trivial) fact that M *is connected, though it is still unknown whether* M *is locally connected — can you travel between nearby points of* M *staying inside* M *without making too long a detour? In 1998, Mitsuhiro Shishikura showed that the boundary of* M *has fractal dimension 2, which means that it is just about as complicated as can be, though it is still not known whether this boundary has positive area.*

This is the fourth volume of Mandelbrot's Selecta*, comprising edited reprints of the author's papers. Largely from the 1980s, these include the series of seminal papers that revealed the magnificence and omnipresence of the Mandelbrot set, together with other papers related to the iteration of functions. Several sections provide an overview of the work along with its scientific and historical background. One chapter has been written specifically to help the non-expert appreciate the rest of the book.*

Much of the material does not require particularly technical knowledge, so the book should be accessible to a wide readership. It provides a fascinating insight into the author's journey of *seeing and discovering as the early pictures of the Mandelbrot set started to reveal a whole new world. It gives a feeling for his philosophy and approach of experimental mathematics — an approach that has changed the way we think about mathematics and science.*

Neue Zürcher Zeitung ◇ SE ◇ Feb 25, 1998 ◇ George Szpiro

[Includes 16 old} articles, many of which were breakthroughs...Unfortunately, reading this book is extremely irritating ...It is the psychogram of a conceited genius...Can be understood perhaps if you consider that success has been denied to him for a long time...He failed to receive the highest award in mathematics, the Fields Medal...and had to be satisfied with the Wolf Prize for Physics.

Physics Today ◇ SE ◇ August 1998 ◇ Nigel Goldenfeld (Physics, U. of Illinois, Urbana-Champaign)

THE STOCK MARKET: BROWNIAN, GAUSSIAN OR MANDELBROTIAN

Most of us would probably be prepared to agree that stock price changes are not Gaussian; nor are they examples of random-walk behavior. However, events such as the 1987 crash, World War I and the crash of 1929 are extreme and properly regarded as outliers. What about business as usual?

The answer, of course, depends on whom you ask. On one hand, Burton Malkiel's well-regarded semipopular book on finance, A Random Walk Down Wall Street *(Norton, 6th edition, 1996), takes its title and its theme from the notion that stock price changes follow a Brownian motion. Virtually every textbook on advanced finance takes the Brownian-motion description as its starting point, and the celebrated Black-Scholes formula for option prices is based upon this description.*

On the other hand, there is Benoit Mandelbrot. No reader of PHYSICS TODAY *can be unaware of the enormous impact made by Mandelbrot's* The Fractal Geometry of Nature *(Freeman, 1982), which introduced many to the notions of fractal dimensions, scaling and self-similarity and spawned a host of coffee-table imitations.*

What is perhaps less well known, however, is that some of Mandelbrot's earliest forays into fractals involved a detailed analysis of the time series for cotton prices in New York. His shocking conclusion, published in 1963, was that the time series was in no way Gaussian. In fact, he argued, the departures from normality could be accounted for by using distribution functions with infinite variance, which are termed L-stable. Mandelbrot examined the convergence in sample number of the variance of the logarithm of the daily price changes and found erratic variation rather than convergence.

Subsequently, his student Eugene Fama (who has himself enjoyed a distinguished career in finance) examined the time series for the 30 stocks in the Dow Jones industrial average, finding no exceptions to the long-tailed nature of the distributions observed.

The implications of these and subsequent findings are profound, yet it is fair to say that the work was practically ignored by economists and practitioners of finance. Even today, the problem of "fat tails" is swept under the rug by the vast majority of financial risk managers, even though the phenomenon is sufficiently widespread and well recognized as to have earned its whimsical name.

Mandelbrot's heirs are primarily physicists who enter the field of finance, recognize the fundamental importance of fat tails and then elaborate on and extend his suggestive results. This is something of an ironic development, as Mandelbrot takes pains to emphasize…and reflects the close intellectual relationship between finance and physics: Brownian motion…was in fact anticipated by Louis Bachelier five years earlier… Elements of Mandelbrot's work in the early 1960s, which superseded Bachelier's analysis just as it was becoming widely accepted, arguably anticipate some of the concepts of scaling and renormalization, which were a focal point of physics during the 1970s.

The concepts of fractional Brownian motion and multifractals, which are still frontier topics of research in physics and academic finance (as practiced by physicists), were introduced by Mandelbrot in the late 1960s and 1970s…

Fractals and Scaling in Finance *is a characteristically idiosyncratic work. At once a compendium of Mandelbrot's pioneering work and a sampling of new results, the presentation seemed modeled on the brilliant avant-garde film* Last Year in Marienbad*, in which the usual* *flow of time is suspended, and the plot is gradually revealed by numerous by slightly different repetitions of a few underlying events.*

As Mandelbrot himself admits in the preface, the presentation allows the reader unusual freedom of choice in the order in which the book is read. In fact, I enjoyed this work most when I read it in random order, juxtaposing viewpoints and analyses separated in time by three decades and making clear the progression of ideas that Mandelbrot has generated. These include the classification of different forms of randomness, their manifestation in terms of distribution theory, their ability to be represented compactly, the notion of trading time, the importance of discontinuities, the relationship between financial time series and turbulent time series, the pathologies of commonly abused distributions (particularly the log-normal) and a catalog of the methods used to derive scaling distributions, both honest and fallacious.

Mandelbrot writes with economy and felicity, and he intersperses the more mathematical sections with frank historical anecdotes, such as the events that led up to his work on cotton pricing and the embarrassment caused by interpreting US Department of Agriculture data for weekly averages as "Sunday closing prices." There are many fascinating asides on a variety of topics, ranging from the importance of computer graphics in science to the distribution of insurance claims resulting from fire damage. In some places, the format of reprinted (but slightly edited) versions of classic papers allows Mandelbrot the surreal luxury of reviewing not only the content, but also the style and presentation of his work. And if all this were not enough, there are guest contributions from Eugene Fama, Paul Cootner and others.

This volume is not intended to be a textbook of modern finance, and it will probably infuriate those seeking a balanced and systematic exposition…but to criticize the volume on that account would be churlish. The reader who is open-minded and prepared to indulge one of our more influential and original thinkers will be amply rewarded.

All in all, this is a strange but wonderful book. It will not suit everyone's taste but will almost surely teach every reader something new. What more can one ask?

Pour La Science ◇ SFE ◇ Janvier 1998 ◇ Ivar Ekeland (U de Paris-Dauphine)

Benoît Mandelbrot … n'est guère modeste, et a d'excellentes raisons de ne pas l'être… Le but avoué de son dernier livre est de montrer l'unité de sa pensée scientifique et de raconter comment il a eu raison avant tout le monde…

Les idées qui animent jusqu'à aujourd'hui l'oeuvre de B.Mandelbrot et les centres d'intérêt auxquels il n'a pas cessé de s'attacher pendant un demi-siècle [sont] la répartition des revenus, l'évolution des cours boursiers, les crues de Nil et la fréquence des mots… B.Mandelbrot insiste à plusieurs reprises sur le fait qu'il avait à peu près tout dit dès le début.

Ses idées sont simples et robustes… Tirons au sort une suite de N valeurs X1, X2, X3…, indépendantes et équiréparties dans l'ensembles des nombres réels. Trois types de situations sont possibles. Premièrement, le cas classique, où les moyennes X1, (X1 + X2)/2 , (X1 + X2 + X3)/3… convergent rapidement vers une valeur finie et certaine (loi des grands nombres), et où l'écart à la moyenne, pondéré convenablement, converge vers une variable aléatoire dont la distribution est représentée par la très fameuse courbe en cloche de Gauss (théorème central limite).

Deuxièmement il y a le cas lent, représenté par exemple par la loi "lognormale".

Troisièmement le cas sauvage, représenté par la loi de Cauchy, dont l'espérance et la variance sont infinies; dans ce cas, les moyennes ont exactement la même distribution que chacun des tirages individuels. aussi loin qu'on aille, on ne vera jamais s'instaurer de compensation entre les différents tirage: la moyenne est aussi incertaine que chacune des épreuves.

Ce sont des cas extrêmes, et il y a entre eux tout un continuum de cas intermédiaires [relatifs à] ce qu'il appelle joliment l' "effet Noé" et l' "effet Joseph".

Ces lois ont d'autres propriétés intéressantes, mais leur intérêt principal est qu'elles fournissent des modèles qui paraissent mieux adaptés que la simple loi normale pour représenter certains phénomènes, comme, par exemple, les crues du Nil ou les cours de la Bourse. On ne peut qu'être frappé, une fois de plus, par le caractère purement visuel des analyses de B.Mandelbrot. Comme il le dit lui-même: "Précisons le rôle des images. J'insiste sur le fait que la capacité d'imiter est déjà une forme de compréhension. Mais j'ai toujours dit, et je *"reconnais" volontiers, que les images doivent nécessairement être suivies de commentaires statistiques objectifs."*

Ce n'est pas ce que la plupart d'entre nous entendent par compréhension. [La] recherche des causes est d'autant plus importante qu'on ne s'intéresse pas à ces trajectoires pour le plaisir, mais pour réaliser certaines opérations de couverture…

Quiconque veut proposer un nouveau modèle doit se soucier de savoir ce que deviennent [les] stratégies de [Black-Sholes]. Ce n'est pas le souci de B.Mandelbrot: il lui suffit d'avoir vu, et de pouvoir montrer. B.Mandelbrot est un mathématicien pur.

Recherche ◇ SFE ◇ Francis Wasserman

L'inventeur des fractales propose ici une … recherche qui se focalise sur les invariances d'échelle, à la base des structures fractales dont il expose ici les principales notions.

L'auteur est amené à définir trois états du hasard: "bénin" (celui de la statistique classique et des lois de Gauss), "lent" ou "sauvage" (ceux de la finance)… Le rôle et l'usage du hasard sont ainsi à la charnière d'une réflexion qui articule un dialogue constant entre l'économie et la physique, entre aléatoire et non aléatoire -cheminement fécond pour les analogies et les rapprochements qu'il a suggérées (crues et mouvements boursiers par exemple).

L'ouvrage présente bien les fondements statistiques des fractales trop souvent masqués par la vulgarisation. A ce titre, la lecture de cette synthèse s'adresse plutôt à un public averti.

Statistica Applicata ◇ SE ◇ **10**, 1988.3, p 504 ◇

Benoit B. Mandelbrot achieved great fame in the scientific world after the publication of "The Fractal Geometry of Nature," which was followed by a variety of books and papers on fractals, a topic that he introduced. Probably this great mathematician is not equally known - except among specialists - for his innovative studies regarding financial phenomena (in particular variations of financial prices and the distribution of personal income), starting with celebrated articles in the early '60s. Reprints of Mandelbrot's papers on the subject constitute about half of this volume, while the other half - for the most part non mathematical - has been written by the author expressly for the publication of this book. Mandelbrot's approach to finance stems from a critique (always sustained and actually strengthened even more) of the traditional use of random walks, of Brownian motion, and of martingales to describe - and, for some, also to explain - price variation or financial markets. His dissatisfaction regarding the use of Brownian motion is based on a detailed analysis of a long series of prices. From these he derives innovative proposals and much more general schemes, which, however, preserve the fundamental stationarity and scaling properties typical of Brownian motion. Mandelbrot's proposal is to allow us to abandon the Gaussianity of Brownian motion, which can describe only mild variations found in many physical phenomena, and to tackle wild variation, which can exhibit genuine discontinuities and turbulences peaked around particular periods.

Statistical Papers ◇ Vol.41, Issue 2, p.245-246, Germany 2000 ◇ Christian Kleiber, Dortmund.

In the last forty years, Benoit Mandelbrot has made his mark in an impressive number of scientific disciplines. To a wider audience he is probably best known for his work on fractal geometry, but he has also contributed substantially to economics and physics, among other fields. The book under review is the first of a multivolume series of Mandelbrot's Selecta, dealing with his work in economics. Future volumes will be devoted to hydrology, turbulence, and other physical phenomena. The general concept is to present some of Mandelbrot's classic papers in a field, along with unpublished work. For the present volume, this means that about half of the material has not been available before. The best-known papers of those included are probably "The variation of certain speculative prices," originally published in the Journal of Business in 1963, and "The Pareto-Lévy law and the distribution of income," originally published in the International Economic Review in 1960. Seven further articles are reprinted here. In addition, one gets a number of expository papers, comments (sometimes on the work of others) and unpublished working papers, mostly from the 1960s. Eugene Fama's well-known article "Mandelbrot and the stable Paretian hypothesis" (Journal of Business, 1963) is included as a guest contribution. Most papers are presented in their original form, however, the titles and the terminology have sometimes been changed. For example, what Mandelbrot used to call *a "Pareto-Lévy" or "stable Paretian" distribution in the 1960s is now called an "L-stable" distribution, and what used to be the "strong Pareto law" is now the "uniform Pareto law."*

The general theme is self-affinity (or power-law behavior, or "scaling") in economics. For example, in empirical finance Mandelbrot's contributions are twofold: first, he observed that quite a few return distributions were incompatible with Gaussianity and suggested, in order to be able to draw on the invariance principles of probability, to replace the normality assumption with that of a non-normal (infinite variance) stable distribution. Second, it turned out that martingale models of the prices themselves could be replaced, at least on a deformed time scale ("trading time"), by something more dependent, a fractional Brownian motion. The first idea, in Mandelbrot's words: 'tail-driven variation,' leads to hyperbolically decreasing tails, the second, 'dependence-driven variation,' to hyperbolically decreasing autocorrelations of the increments of such processes. It took some time to nest these two features within a more general model, the "multifractal model of asset returns," which has only recently been worked out in detail and is still unpublished. Chapter 6 of this book nevertheless provides the basic ideas.

As Mandelbrot states in the preface, his methods of investigation are those of "practicing theoretical and computational physicist." Hence, the presentation is frequently not mathematically rigorous - this is, however, amply compensated by a wealth of ideas. For example, it is impressive to see how many results in e.g. Samorodnitsky and Taqqu's (1994) "Stable Non-Gaussian Random Processes" have been inspired by Mandelbrot's work of the 1960s and 1970s.

In spite of the title, this book is not exclusively concerned with financial economics. It also contains some work on the distribution of income and the size distribution of firms, "Fractals and Scaling in Economics" would be a better title. In view of the recent boom in empirical finance, the publisher apparently hopes to boost sales this way.

Of the newly added material, I particularly enjoyed chapter 4, "Sources of inspiration and historical background," where Mandelbrot finds scaling, apart from economics and physics, in the laws of allometry, in the work of Jonathan Swift, or in a line of William Blake's. Also, his comments on whole branches of the literature are always entertaining: the most popular brand of nonlinear models for stock returns, for example, is dismissed on p.44 as "patchworks of quick fixes called ARCH models."

To sum up, this is a most useful collection of Mandelbrot's work economics, it provides an excellent starting point for anybody interested in the origin of many current topics in empirical finance or the distribution of income.

La Tribune ◇ SEF ◇ Lundi 2 Février 1998 ◇ Lysiane J. Baudu

LE CHOU-FLEUR ET LES MARCHES : Le mathématicien Benoît Mandelbrot, presque autodidacte, est le père de la géométrie fractale. En l'appliquant aux marchés financiers, il fait comprendre leur évolution erratique**.**

Benoît Mandelbrot [est] célèbre dans le monde des mathématiciens pour ses théories inédites sur la géométrie fractale, qui fait comprendre l'irrégularité des formes, comme celle des flocons de neige ou des réseaux de rivières. Des théories qu'il cherche à mettre au service de la finance, depuis quelques années, pour comprendre l'évolution erratique des cours de la Bourse. Mais pour expliquer ce concept, il s'aide d'un chou-fleur … et explique que ce légume [a une] surface ruguese. "Alors que l'ouïe, le chaud, le goût, ont été très bien étudiés par la science, le sens du rugueux est resté quasiment inexploré," remarque-t-il. Et il n'y a pas que le chou-fleur dont la surface est rugueuse, les prix des actifs financiers aussi, Cette montagne de science, du haut de son 1,90 mètre, enfourche son cheval de bataille. Pour expliquer … que l'évolution des cours des actifs est discontinue, que les prix peuvent décaler brutalement, et non pas de façon graduelle et continue. Comme le beau temps et le mauvais temps en somme… Il ne s'agit en aucun cas de prévoir un krach, l'homme est trop réaliste pour cela. Il essaie simplement de mesurer la "rugosité" des cours, et d'en ramasser la complexité en une formule simple...

Les théories de Benoît Mandelbrot sont célèbres dans le monde entier. "De grandes banques américaines avaient débloqué un gros budget, une équipe de plusieurs dizaines de personnes pour travailler sur le chaos, raconte un professionnel, maintenant ce sont effectivement les théories fractales qui sont à la mode." Même si certains ne croient pas *franchement au bien-fondé de la théorie fractale appliqué à la finance, ils restent impressionnés par les recherches et le savoir du mathématicien de Yale.*

Wirtschaftsinformatik (BRD) ◊ SE ◊ **41**, 1999.1, p 95

Zentralblatt der Mathematik ◊ SN ◊ Yimin Xiao

For self-similar fractals, the most important aspect is the measure of roughness or irregularity called fractal dimension. Self-affine fractals intrinsically are far more complicated in many ways. In this book, Mandelbrot intends to show that self-affinity helps apprehend and organize the baroque wealth of structure found in nature. This book is mainly devoted to two broad topics: 1/f noises and multifractals, which arose independently and developed as two separate subjects and their histories carry very distinct flavors. Indeed, 1/f noises are of primary concern in signal processing and applied physics; while multifractals began in the study of turbulence and dynamical systems, and also in finance as shown in M1997E. Nevertheless, Mandelbrot defends successfully the view that multifractals and 1/f noises are actually intimately interrelated -- they belong to the broad and unified mathematical notion of self-affine fractal variation which ordinarily implies uniform global statistical dependence, and they are best viewed as being distinct aspects of a wild rather than mild random or non-random phenomenon. An 1/f noise is defined as having a spectral density proportional to f^{-B}*, where B is a constant which is called the spectral exponent. Since 1/f noises with the same spectral exponent can take any of many different forms, understanding such a noise demands more than just the spectrum. The link of 1/f noises to self-affinity makes it possible to study 1/f noises by introducing a formalism made of scaling laws, renormalization and fixed points. By focusing on three forms of 1/f noise: fractal dustborne noises, multifractal noises, and Gaussian 1/f noises… Mandelbrot sketches a broad analytic method for discriminating between those various possibilities. The excellent term "multifractal" was chosen by Frisch and Parisi in 1985. But constructive and rigorous approach to multifractals as physical models was developed in 1970s by Mandelbrot (see comments about Part IV below). The multifractal formalism centers on two mutually related functions* $\tau(q)$ *and* $f(\alpha)$ *(via the Legendre transform), each can be introduced in several closely related, but non-equivalent ways. Mandelbrot's original approach was based on Cramèr's theory of large deviations.*

This book consists of four parts. Part I provides newly written introductions to the book which relate diverse models and themes to one another. In particular, Chapter N1 gives a panorama of grid-bound recursive self-affine constructions that allow for a great diversity of behavior, including simplified cartoons for Wiener Brownian motion, fractional Brownian motion, Lévy flight, 1/f noises of several distinct kinds and multinomial multifractals, with a view towards further research topics. Chapter N2 describes how the reprints classified into Parts II, III, and IV fit together historically and examines conceptual connections and relevant historical events. Part II is primarily concerned with self-similar unifractal models in nature. The underlying constructions are random Cantor sets and Lévy dusts. Chapter N6 reproduces the important paper of Berger and Mandelbrot (1963) which proposed a new mathematical model based on a Cantor dust to describe the occurrence of errors in data transmission on telephone lines and was the first to interpret the previously esoteric notion of fractal dimension as a fundamental physical quantity. This model is improved and generalized in Chapter N7, in which the concept of conditional stationarity is introduced. Part III concerns the fluctuations called 1/f noises that are "dustborne", that is, vary when time belongs to a Cantor or Lévy dust. Such noises are called "sporadic" or "absolutely intermittent". It argues that the Wiener-Khintchine spectral theory and even the conventional theory of stochastic processes are not enough in investigating such wild noises and begins constructing two generalizations: conditional spectral analysis and sporadic processes. Part IV addresses multifractal measures and turbulence. The centerpiece reproduces [J. Fluid Mech. 62, 331-358 (1974)]. Its objective was to show that Kolmogorov's "third hypothesis" -- the probability distribution of the average of the dissipation in intermittent turbulence is lognormal -- was untenable. As it turned out that it is of broader impact: it was the first paper to investigate the concept of random multifractal measure. The random fractals in Chapter N15 are created by multiplicative random cascades. Mandelbrot's approach relies upon Cramèr's large deviation theory, while in some later investigations, *including [Halsey et al], the approach is exclusively analytic. This part also contains the important paper of J.-P. Kahane and J. Peyrière [Adv. Math. 22,131-145 (1976)], which solves several conjectures about the Mandelbrot's canonical model for turbulence and later has inspired a lot of further research in mathematics.*

This volume of Mandelbrot's Selecta is a major contribution to the understanding of wild self-affine variability and randomness. It is also a mine of other new ideas which are of use to diverse scientific communities from physics, pure mathematics, to finance. Finally, for more topics on multifractals such as multifractal functions, fractional Brownian motion of multifractal intrinsic time and their relevance to finance….

Zentralblatt der Mathematik ◇ SH ◇ Yimin Xiao

This book contains the author's works in areas ranging from statistics, mathematics, physics, hydrology to economics and finance that first appeared from 1965 to 1988, as well as more than 200 pages of new material written especially for this volume. Most of this book deals with random functions that vary in continuous time and take continuously distributed values. In many cases, they follow the Gaussian distribution. The most important random function in this book is the fractional Brownian motion. A large portion of the book is related to the geometric studies of fractional Brownian motion.

Mandelbrot chooses the letter H to denote this volume among other Selecta because, as he explains, "The self-affinity exponent entered science through my response to remarkable findings of the hydrologist Harold Edwin Hurst". The letter H also refers to "Hölder exponent", a mathematical term characterizes the local behavior of a function. Volume H has many close connections to the topics of Selecta E and N. Some of the links are described in Chapters H1 and H30. Compared to these previous volumes, Volume H has a higher level of mathematics and contains many theorems, problems and conjectures phrased in mathematical terms. Together with Selecta E and N, this volume further develops many aspects of the author's famous Essay [The Fractal Geometry of Nature, Freeman (1982)]. Volume H begins with Chapter HO which is an overview of fractals and multifractals. It tackles broad issues and answers diverse questions. Mandelbrot shows that fractal geometry has one focus in mathematics and another in the broadly based discovery that scale-invariant roughness is ubiquitous (both in Nature and in man-made structures) but can be handled quantitatively.

Fractal geometry is a study of scale-invariant roughness, and it is a quantitative and organized new "language of shape", a "toolbox" of statistics and data analysis, geared toward the study of wild randomness and variability. Together with Chapter H8, this introduction describes the far-reaching and highly relevant historic roots of fractal geometry, the author's principle contributions to various scientific areas related to this book, the content of fractal geometry in 2001 and his view towards further research topics. The rest of this book consists of seven parts. Parts I and II consist of specially written introductions to the book which relate diverse models and themes to one another. The unifying concept is self-affinity, which is surprisingly rich and involved and is still under development.

Chapter H1 approaches the concept of self-affinity by concentrating on a quite narrow family of self-affine curves constructed recursively from an initiator and a generator. It is shown [see also the long foreword of Chapter H24] that small changes in the generator can have spectacular affects on the resulting self-affine curve. Chapter H2 discusses several different notions of self-affinity. As a particularly important example of self-affine Gaussian fractals, Wiener Brownian motion (WBM) is treated in Chapter H3, in which its structure and fractal dimensions are investigated. Chapter H3 not only discusses the classical results on the Hausdorff-Besicovitch dimensions of the trails and records of WBM, but also describes several old and new conjectures on the Hausdorff-Besicovitch dimension of the hulls of the Brownian and percolation clusters. Exciting new developments include the recent mathematical proof of Lawler, Schramm & Werner (2000) on Mandelbrot's famous 4/3-conjecture about the Hausdorff dimension of the Brownian cluster [M1982FGN, Plate 243], and the proof of Smirnoff (2001) about the corresponding 7/4 and 4/3 conjectures on percolation clusters. These results show that, contrary to the impression that WBM is a well-understood, mature topic, the fractal-dimensional properties of WBM prove to be multiform, complex and subtle in many ways.

Chapter H4 discusses the Weierstrass family of functions. Even though it is only partly concerned with Gaussian fractals, its topic is highly relevant to this book's goals. The original Weierstrass functions serve as examples of functions that are continuous but not differentiable. These "old" functions are modified and extended (e.g., adding "subharmonics" and allowing random phases) to get "new Weierstrass functions" with increasingly rich invariance. They are useful in diverse ways. Chapter H5 proposes a classification of diffusion processes into isodiffusive and heterodiffusive processes based on their R/S local (also called short-term) and global (also called long-term) dependence. The existence of global dependence is due to either of two causes, alone or together: global dependence which is labelled the "Joseph effect", and fat (heavy) tailedness, which is labelled the "Noah effect".

H6 discusses transformation between self-affine functions and stationary processes, using a logarithmic time clock. In the Wiener Brownian motion case, this time change gives the Ornstein-Uhlenbeck process.

Chapter H7 is on empirical power-law behavior, 1/f noise and their connections to self-affinity. This chapter is closely related to Chapter N3 of Selecta N (1999). Chapter H8 tackles topics close to those of the overview chapter HO with more descriptions of the author's personal involvement. It contains "a little-known historical episode concerning random walk, Brownian motion and the role of the eye in assisting scientific understanding" and a summary of the author's principle contributions to the topics of this book. H8 also brings together (in chronological order) some personal recollections of improbable encounters with fields, collaborators, referees and editors.

Parts III to VII of this volume consist of reprints of the papers of the author and his co-workers on the topics between 1965 and 1988, with added forewords, annotations and historical remarks. Part III consists of Chapters 9 & 10 and can be regarded as an easier introduction to the subject of this book. Chapter H9 is the oldest paper reprinted in this volume. It links Hurst's power law with self-affinity, and introduces fractional Brownian motion (without the name). H10 proposes a family of statistical models of hydrology which account for the Noah and Joseph effects in studies of precipitation.

Parts IV and V concern fractional Brownian motion, fractional Brownian surfaces and their usefulness in statistical modelling. Chapter H11 reprints the famous paper of the author with Van Ness (1968) in which they defined fractional Brownian motion and investigated its moving average representation, self-affinity and long range dependence. Chapters H12 and H13 test the quality of fractional noise as a model of reality and to estimate its parameters. Two approaches are the R/S analysis and spectral analysis, of which the former is discussed in more detail in Part VII. Chapter H14 is complementary to H11 [M & Van Ness (1968)]. H15 describes a fast fractional Gaussian noise generator. Chapters H17-20 of Part IV deal with fractional Brownian surfaces [multidimensional-time extensions of fractional Brownian motion], their mathematical constructions, self-affinity and applications in modelling turbulence and the Earth's relief.

Part VI intermingles the themes of self-affinity, various fractal dimensions and multifractals. The results on fractal dimensional properties of self-affine sets show that, in contrast with the case of a self-similar sets where there is a unique fractal dimension, several distinct dimensions are needed for characterizing the structures of self-affine fractals. Mandelbrot believes that the richness and complexity of the study on self-affine sets are not purelymathematical but reflect the richness and complexity of nature.

Chapter H21 serves as an introduction on the main points of Part VI. H22 discusses the local and global mass/box dimensions and the gap dimension of self-affine fractals. H23 deals with self-affine fractal curves and shows that, "walking a divider" along a curve yields a local and a global values that may be different from the mass/box dimension of the curve. H24 discusses a result of C. McMullen [Nagoya Math. J. 96, 1-9 (1984)] which shows that the Hausdorff-Besicovitch dimension of a recursively constructed self-affine fractal may be strictly smaller than its local box dimension, and the implications of this result to new developments in fractal geometry. One of the implications is, as Mandelbrot suggests, the special standing of the Hausdorff-Besicovitch dimension in fractal geometry may have lost.

Part VII studies R/S analysis systematically. It contains reprints of the two papers with J. R. Wallis [H25 & 27] which discuss the foundation of the R/S method in depth and address issues *in hydrology and geophysics, a mathematical paper of himself [H26] on limit theorems for the self-normalized bridge range, and papers on applications of R/S analysis to studying the secular motion [H28], linguistics [H29], economics and finance [H30].*

To summarize, Volume H of Mandelbrot's Selecta is a major contribution to the understanding of wild self-affine variability and randomness. I believe that, similar to M1982FGN, it will have a great impact on future research in mathematics, statistics, physics and other applied areas.