REMARKS ON STATISTICAL THERMODYNAMICS
(Discussion of a paper by R.E. Collins)

Symposium on Critical Review of Thermodynamics (Pittsburgh PA).
Edited by Edward B. Stuart, Benjamin Gal-Or
& Alan J. Brainard. Baltimore, MD: Mono Book Corp., 1970, 32-34.

B. Mandelbrot: Though I have not yet had the leisure to study Prof. Collins' paper in depth, three sorts of remarks seem to be in order concerning, respectively, (A) his choice of formalism, (B) the application of this formalism to physics, and (C) the possible application of the thermodynamic formalism outside of physics.

(A) The concept of "embedding" is reminiscent of the concept of "sufficiency," as defined in the mathematical probability and statistics. The great importance of sufficiency in thermodynamics, and in particular the possibility of deriving Gibbs' distribution on this basis, have been observed in my Note in the Paris *Comptes Rendus* for 1965 (Vol. 243, p. 1835) and developed in detail (and in English) in my paper "The Role of Sufficiency and of Estimation in Thermodynamics," *Annals of Mathematical Statistics*, Vol. 33 (1962), pp. 1021–1038.

(B) To select a formalism is only a first step. The next step is to identify the properties that make a given formalism applicable to matter, where the principal among such properties are called *principles of thermodynamics*. If, as is widely believed, the main concepts of thermodynamics are three in number, namely, the energy, the temperature, and the entropy, one needs at least three principles, ordinarily numbered (somewhat perversely) as the first, the zeroth, and the second. Professor Collins has not told us, I believe, which form he favors for these principles.

One form of the zeroth principle I favor expresses the following idea: *It is the nature of thermal equilibrium that N systems* $S_1, \cdots, S_N$ *that are in thermal equilibrium with each other and any "external" system* S_0, *will remain in thermal equilibrium when they are isolated from* S_0. To express this mathematically, the joint probability distribution of the random energies $U_1, \cdots, U_N$ of the systems $S_1, \cdots, S_N$ must be evaluated under two different conditioning assumptions, and then one must assume that these two conditional probability distributions are identical. (a) The first condition is that the combined system $S_1 + \cdots + S_N$ *is* in contact with S_0 and happens to include (as a result of the fluctuations of energy between itself and S_0) an energy equal to $U_1 = \cdots U_N$. (b) The second condition is that the combined system $S_1 + \cdots + S_N$ *is not* in contact with S_0 but still includes the same energy $U_1 + \cdots + U_N$. It can be shown that the identity between the above probability distribution naturally introduces the formalism of statistical sufficiency mentioned in (A).

To obtain the Gibbs distribution and the concept of "canonical ensemble" from the above assumption about thermal equilibrium, it is necessary to define a heat reservoir. A heat reservoir is usually defined by a limit process, as being an extremely large external system S_0, but I rather prefer to characterize a heat reservoir directly, as being a system S_0 such that, when S_0, S_1, $\cdots$ and S_N are in mutually thermal contact, the random energies U_1, $\cdots$, U_N are statistically independent.

The basic idea, that breaking thermal contact does not affect thermal equilibrium, has been independently proposed by Szilard in 1925 and G. N. Lewis in 1931, but despite both authors' fame, these two papers were "lost" from the literature. I hope that the only cause of such neglect was that neither author provided a suitable formalism to go with his informal physics. If indeed there is no more fundamental reason for its neglect, my own independent rediscovery of the Szilard–Lewis approach may hope to be better received, since it arose accompanied by a full-fledged "sufficient statistics" formalism, as already mentioned under (A). For a description of the resulting axiomatic, see, in addition to the references given under (A), my paper "On the Derivation of Statistical Thermodynamics From Purely Phenomenological Principles," *Journal of Mathematical Physics*, Vol. 5 (1964), pp. 164–171. Further study of the "statistical phenomenological" approach is found in the paper by Laszlo Tisza and Paul M. Quay: "The Statistical Thermodynamics of Equilibrium", *Annals of Physics*, Vol. 25 (1963), pp. 48–90 (Reprinted in Tisza's *Generalized Thermodynamics*, The M.I.T. Press, 1966) and in Tisza's paper in the present Symposium.

(C) Now let us turn to the proposed applications of the thermodynamic way of thought outside of physics, in fields like economics or sociology. To take one example, many authors claim to have explained the Pareto distribution of personal income by thermodynamics. They argue that energy can be "reinterpreted" in this context by the logarithm of income, and then form the most likely distribution of income (which is also the only "embeddable distribution," in Collins' terminology). It is true that, in a formal sense, this approach "generates Pareto's distribution," but this does not suffice in my opinion to make us consider this approach as a genuine application of the thermodynamics way of thinking. The difficulty is, simply, that the quantity *"log income" is not additive* so that there is no concrete interpretation to the step in which the *average log income* is fixed, while one forms the most likely distribution of the log income of an individual. The usual justification for considering log income goes through "proportional effect" arguments, according to which it is the relative not the absolute income differentials that count. Whether this motivation is sound economics is beside the point; it does not justify calling log income a "reinterpretation of energy."

In my opinion, my theory of word frequencies (see my discussion of Professor Ramsey's paper in this volume) is the only successful application of thermodynamics thinking in a social science. The physicist's attitudes are extremely useful in social science modeling (as I hope to have demonstrated elsewhere), but the techniques that have been successfully transposed are not, in general, those of thermodynamics.