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Detail from a Mandelbrot set.

Father of fractal complexity

Benoit B Mandelbrot, Sterling Professor of Mathematical Sciences, Yale University; IBM Fellow Emeritus, IBM Thomas J Watson Research Center.

Benoit Mandelbrot might be called a complex character. He is a self-proclaimed maverick whose multi-disciplinary approach has created a whole new field of mathematics, and who, after long years of solitary and often abstruse work, became a household name as his work was embraced by scientists, artists and the general public.

Mandelbrot is widely hailed as the father of fractal geometry—a new geometry of roughness with the power to describe a vast array of natural and manmade phenomena from the shape of coastlines to daily share price movements. The key idea is that 'fractals'—a word coined by Mandelbrot—are shapes which have the same general structure on any scale.

His work has been recognized by numerous awards, including the Wolf Foundation Prize for Physics in 1993 and the 2003 Japan Prize alongside chaos theory pioneer James Yorke for outstanding achievements in the science and technology of complexity.

Mandelbrot was born in Poland in 1924 to a Lithuanian family, and moved to France in 1936 to escape growing political strife. Completing his schooling in Paris after the war, he took top marks in mathematics but looked in vain for a way to combine this with his extraordinary gift for shape and visualization. He first came to the US in 1948 when the École Polytechnique arranged a scholarship to Caltech. Returning to Paris, he completed a PhD dissertation combining mathematical linguistics and statistical thermodynamics, an early sign of his distaste for the boundaries between academic disciplines.

He crossed the Atlantic again in 1953 to study at the Institute for Advanced Study in Princeton, as the last post-doctoral fellow sponsored by John von Neumann, the pioneer of game theory. After four years back in Europe, Mandelbrot joined the IBM Thomas J Watson Research Center, New York, as a research staff member in 1958. He remained there for 35 years, and was made an IBM Fellow with the freedom to pursue his own interests in 1974.

By the early 1960s, Mandelbrot found his attention turning to some basic questions of financial economics. 'Since my PhD in 1952, I'd been working on power-law distributions, which at that time were extremely unpopular and today are very much in fashion', he recalls. 'I was soon attracted to income distribution and then to price variation. These topics were very far from my training in mathematics and physics, but very much in line with a belief that important questions should be studied without too much concern for the boundaries between disciplines.

'I went into finance because it was of interest, and my position let me go in. In the 1960s I put down the foundations of one part of what is now called econophysics, first the distribution of price changes and then their long-term variation.'

Early computers were then starting to allow economists to analyse long-term price changes, usually based on the Brownian model developed by Louis Bachelier at the turn of the century.

'Everyone tried to apply existing techniques to the newly available data on price variation, and a horrendous mismatch was observed', Mandelbrot says. 'The techniques of analysis developed over many decades were very good in other contexts but proved to be very inapplicable to price variation.'

The basic assumption was that price variations followed the Bell curve or Gaussian distribution, in which extreme changes were vanishingly rare. But as anyone who follows price movements knows all too well, ten-sigma changes or larger can be seen every day in the financial pages. Over a year, the most important changes can occur in just a handful of days, but models based on Gaussian distributions either assumed large price changes to be non-existent, or damned them as anomalies to be dealt with by some other theory.

'Being ambitious, I wondered if this was necessarily so, and looked for a model or scenario which would include both the small price changes in the middle of the Bell and larger ones in the long tails of the Bell', Mandelbrot says. 'I discovered such a model, tested it for many kinds of data and published several landmark papers on the subject.'

In a groundbreaking paper in 1963 [1], Mandelbrot showed that both the large sudden changes and the 'background noise' of the small-scale changes of Brownian motion both fall on a stable Lévy power-law distribution. While this addressed the question of long-term variance, there was still the issue of changing price volatility. Mandelbrot first approached this with the concept of fractional Brownian motion [2], and then, with Howard Taylor, with the new notion of intrinsic trading time [3], a random distortion of clock time to account for changes in volatility.

'This work became a great sensation because it was different from anything that anybody had thought of in this field', Mandelbrot recalls. 'It opened up an entirely different way of looking at variability, and many people wanted to prove me wrong.'

In the public eye

With the growing popularity of the Black–Scholes model, which failed to address the issues addressed by his own concepts, Mandelbrot moved away from finance to deeper questions of complex systems. 'I became interested in turbulence and, in the early 1970s, realized with great pleasure that my techniques to deal with prices and turbulence were closely related', he says. 'By adding additional problems which I also became interested in, I realized I had started a new approach to the study of phenomena. That's when I coined the word 'fractal'.'

The basic concept of fractal geometry—self-similarity—is now familiar throughout the worlds of science and the arts, echoing as it does the alchemical dictum of 'as above, so below'. The world is full of systems that look similar on any scale, from coastlines and clouds, to galactic clusters and blood vessels, as illustrated in what was the first of a series of lavishly illustrated and best-selling books [4].

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'I put pictures at the front of my books because I faced a very hard job of convincing', Mandelbrot says. 'While the bulk of science had been studying smooth phenomena, I was studying rough phenomena. People had to see it to believe it.'

For many people, this was the first time that science and mathematics had been made beautiful. This visual sense is something that science had long been lacking, Mandelbrot believes.

'Since I was 20, the hard sciences had become almost completely devoid of pictures', he says. 'I happen to be entirely eye-centred—by every standard, I'm among the most extreme in the quickness and accuracy of what I see. This is why I went away from becoming a normal mathematician or physicist to search for areas where my eye could serve me. In the mid-1960s I convinced some friends at IBM to put together some very primitive equipment to make pictures of the work I was doing. Suddenly not only did that work become convincing, it became attractive, even when I was using the very early graphics. It was an enormous pleasure, and I also believe the great majority of lay people are very much more attracted to visual than to algebraic evidence.'

By the mid-1980s, the Mandelbrot set—a fractal image of complex numbers obeying a simple algebraic rule—had become something of a pop icon. Mandelbrot half-jokingly compares the popularity of his images to that of a beautiful woman who also wants to be known for her brains. 'I didn't slave to achieve them because I wanted to do nice pictures, but because I wanted to convince people that my theories were right', he says. 'I have been very lucky since work that is in a certain sense very austere, was helped by pictures that in many ways are almost baroque.'

Multifractals and beyond

Mandelbrot returned to finance in the mid-1990s, as a new generation of economists embraced his ideas. 'My very clear plan was that as soon as the obvious difficulties with Black–Scholes were recognized, I would just jump back in and resume my work, which I did', he says. 'At the same time, a very large number of people jumped in and now a very large community has adopted my thinking completely and develop it on their own, but I do have certain skills that allow me to also continue on my own.'

The new work, which Mandelbrot says was a 'gigantic advance' on his 1960s work, is based on multifractals, a concept he first developed in the late 1960s to handle turbulence and at the time won little attention in the finance community. In the meantime, multifractals had been deployed in the physical sciences to describe the distribution of energy and matter in systems such as stellar matter and minerals.

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'Multifractals are a very powerful idea that I don't think has been developed enough so far', Mandelbrot says. 'It was a novelty of a high order.'

The multifractal scenario for price changes was detailed in a 1997 book [5], combining long power-law tails and long power-law dependence together with trading time, the key concept allowing the application of multifractals to financial markets. In one form of this new model, price is a fractional Brownian function of a trading time, which is itself is a non-decreasing multifractal function of clock time.

The book also introduced a versatile family of functions that provide a simplified model of the main features of financial prices. Mandelbrot called these 'cartoons', as they make the concepts easily understandable through simple visual representation.

'Once the understanding has spread in wider circles that fractals handle roughness in a way that is controllable, with its own techniques, many things need no longer be done by inspired guesses but by rational design', he notes. 'Fractality, to my delight, and I must confess surprise, is found everywhere: sometimes something quite unimportant, sometimes something quite central. The cases where it is quite central include prices, where I think every method of describing price variation is either straight out of my repertoire, or an approximation to it.'

Now in his late 70s, Mandelbrot continues to publish, and in 1999 became Sterling Professor of Mathematical Sciences at Yale. But with the growth of interest in multifractals he now feels that the area is becoming too crowded for his maverick tastes.

'I do not like to be part of any crowd and try to keep to issues where I have a competitive advantage.' He is currently finishing a paper which he says will 'grab attention'. His ever-growing band of followers would expect nothing less.

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For a detailed introduction to Mandelbrot's recent work, see:

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