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Phase transitions on fractals: I. Quasi-linear lattices

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Abstract. Magnetic spin models and resistor networks are studied on certain self-similar fractal lattices, which are described as 'quasi-linear', because they share a significant property of the line: finite portions can be isolated from the rest by removal of two points (sites). In all cases, there is no long-range order at finite temperature. The transition at zero temperature has a discontinuity in the magnetisation, and the associated magnetic exponent is equal to the fractal dimensionality, D. When the lattice reduces to a non-branching curve the thermal exponent $\nu^{-1} = y$ is equal to D. When the lattice is a branching curve, y is related, respectively, to the dimensionality of the single-channel segments of the curve (for the Ising model), or to the exponent describing the resistivity (for models with continuous spin symmetry).

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