

SPECIAL RELEASE ON MANDELBROT'S CONTRIBUTIONS IN FINANCE

NEW HAVEN, CONN. Benoit Mandelbrot, Sterling Professor of Mathematical Sciences at Yale University and the "father of fractals," shared the 2003 Japan Prize for Science and Technology.

In economics, Mandelbrot has been best known since the early 1960s as one of the pioneers in studying the variation of financial prices. Even before Bachelier's Brownian model became widely accepted in academia, he was the first voice warning against that model's pitfalls. He pointed out that two features of that model, to be described below, are unacceptable. Having discovered that each involves an empirical power-law distribution, he modeled both distributions, first separately and then jointly, on the basis of the concept of fractals. Under the term "scaling in finance," this concept is the topic of Chapter 38 of his 1982 book "The Fractal Geometry of Nature." Thus, scaling became important in finance before it became important in physics. Recently, Mandelbrot greatly extended this work, and it is gaining increasing influence on the work of others.

Altogether Mandelbrot's contributions to finance fall into three main stages.

He was the first to stress the essential importance, even in a first approximation, of large variations that may occur as sudden price discontinuities. The Brownian model is unjustified in neglecting them. They are not "outliers" one can safely disregard or study separately. To the contrary, their distribution is much more important than that of the "background noise" constituted by the small changes of Brownian motion. He followed this critique in [33] by showing in 1963 that the big discontinuities and the small "noise" fall on a single power-law distribution and represented them by a scenario based on Levy stable distributions. He and Taylor introduced in 1967 [51] the new notion of intrinsic "trading time." In recent years, fractal trading time and his 1963 model have gained wide acceptance.

Secondly, Mandelbrot tackled the fact that the "background noise" of small price changes is of variable "volatility." This feature was ordinarily viewed as a symptom of non-stationality that

must be studied separately. To the contrary, Mandelbrot interpreted this variability as indicating that price changes are far from being statistically independent. In fact, for all practical purposes, their interdependence should be viewed as continuing to an infinitely long term. In particular, it is not limited to the short term that is studied by Markov processes and more recently ARCH and its variants. In fact, it too follows a power-law side of dependence. He followed this critique and illustrated long-dependence by introducing in 1965 [38] a process called fractional Brownian motion which has become very widely used.

Thirdly, he introduced the new notion of multifractality that combines long power-law tails and long power-law dependence. Early on, his work was motivated by the context of turbulence, but he immediately observed and pointed out that in 1972 [66] the same ideas also apply to finance. After a long hiatus while he was developing other aspects of fractal geometry, he returned to finance in the mid-1990s and developed the multifractal scenario theory in detail in his 1997 book "Fractals and Scaling in Finance" (Springer). He introduced "cartoons" that realize long tails and long dependence and a very simple process understandable to experts and beginners alike.

One early obstacle to the acceptance of Mandelbrot's work was its heavy reliance on computers; this obstacle has, of course, long vanished. The fact that statistics provided no ready-made tools to handle long tails and long dependency was another obstacle, but largely only a source of delays. The concept of scaling invariance used by Mandelbrot started by being perceived as suspect, because at that time other fields did not use it. However the period after 1972 also saw the growth of a new subfield of statistical physics concerned with "criticality." The concepts used in that field are similar to those Mandelbrot had been using in finance. His early terminology, has been replaced by those introduced by physics, like "scaling" and "renormalization fixed-points."

Be that as it may, the originality of Mandelbrot's work had been recognized all along. This started in 1964 when P.H. Cootner called it "revolutionary". Cootner also raised many questions that have all been answered. Forty years later, Mandelbrot's "revolution" is bearing fruit in many diverse ways.