LOUIS BACHELIER (1870-1946)

The beginnings of the theory of Brownian motion are worth knowing and are touched upon in the next chapter. However, physics might have been preceded in this context by mathematics—and also, in a most unusual sequence of events, by economics.

The fact is that a truly incredible proportion of the results of the mathematical theory of Brownian motion had been described in detail in a doctoral dissertation in the mathematical sciences, defended in Paris on March 19, 1990, five years before Einstein. The precursor was Louis Bachelier (*Dictionary of Scientific Biography*, **I**, 366-367).

Sixty years later, Bachelier 1900 received the rare compliment of an English translation, with extensive comments. However, it started badly: the committee that examined it was not overly impressed and gave it the unusual and near-insulting *mention honorable.* At that time no one stood for the French doctorate unless he foresaw an academic opening and felt sure of receiving the required *mention très honorable.*

It is not surprising therefore that this dissertation had no *direct* influence on anyone else's work. Bachelier, in turn, was not influenced by anything written in this century, even though he remained active and published (in the best journals) several papers filled with endless algebraic manipulations. In addition, his popular book, Bachelier 1914, enjoyed several printings and is still worth reading. It is not to be recommended to just anyone, both because its subject matter has changed profoundly, and because it is not clear whether its short sentences are meant to summarize established knowledge or outline problems yet to be explored. The cumulative effect of such ambiguity is rather disconcerting. Only very late in life, after repeated failures, was Bachelier finally appointed to a University professorship, in the tiny University of Besançon. As of 1982, the trace he left reduced to odd scraps of recollections by students and colleagues, and not a single photo. Since then, a reader of this book sent me a snapshot photo and the centennial in 2000 provoked a more diligent search. Also, by then, his

work had long become obsolete and he became the unwitting "godfather" of strictly academic exercises.

In view of this slow and mediocre career the posthumous fame of his dissertation makes him an almost romantic personality. Why the sharpness of this contrast?

To begin with, his life was much affected by a blatant mathematical error. Its truly strange story is told in Lévy 1970 (pp. 97-98) and in greater detail in a letter Paul Lévy wrote me on January 25, 1964.

"I first heard of him a few years after the publication of my *Calcul des Probabilités,* that is, in 1928, give or take a year. He was a candidate for a professorship at the University of Dijon. Gevrey, who was teaching there, came to ask my opinion of a work Bachelier published in 1913 (*Annales de l'Ecole Normale*). In it, he had defined Wiener's function (prior to Wiener) as follows: In each of the intervals $\&lbt.n\tau,(n+1)\tau\&rbt.$, he considered a function $X(t \mid \tau)$ that has a constant derivative equal to either $+v$ or $-v$, the two values being equiprobable. He then proceeded to the limit (v *constant*, and $\tau rar 0$), and claimed he was obtaining a proper function $X(t)$! Gevrey was scandalized by this error. I agreed with him and confirmed it in a letter which he read to his colleagues in Dijon. Bachelier was blackballed. He found out the part I had played and asked for an explanation, which I gave him and which did not convince him of his error. I shall say no more of the immediate consequences of this incident.

"I had forgotten it when in 1931, reading Kolmogorov's fundamental paper, I came to 'der Bacheliers Fall.' I looked up Bachelier's works, and saw that this error, which is repeated everywhere, does not prevent him from obtaining results that would have been correct if only, instead of $v = constant$, he had written $v = c\tau^{-\frac{1}{2}}$, and that, prior to Einstein and prior to Wiener, he happens to have seen some important properties of the so-called Wiener or Wiener-Lévy function, namely, the diffusion equation and the distribution of $\max_{0 \le \tau \le t} X(t)$.

"We became reconciled. I had written him that I regretted that an impression, produced by a single initial error, should have kept me from going on with my reading of a work in which there were so many interesting ideas. He replied with a long letter in which he expressed great enthusiasm for research."

That Lévy should have played this role is tragic, for his own career, as we will see very soon, also nearly foundered because his papers were not sufficiently rigorous.

We now reach the second and deeper reason for Bachelier's career problems. It is revealed by the title of his dissertation, which I have purposefully not yet mentioned: "Mathematical theory of speculation." The title did not by any means refer to (philosophical) speculation on the nature of chance, rather to (money-grubbing) speculation on the ups and downs of the market for consolidated state bonds (*"la rente"*). The function $X(t)$ mentioned by Lévy stood for the price of these bonds at time t.

The professional difficulties that Bachelier was to experience as a result were foreshadowed in the delicately understated comment by Henri Poincaré, who wrote the official report on this dissertation, that "the topic is somewhat remote from those our candidates are in the habit of treating." One may argue that Bachelier should have avoided seeking the judgment of unwilling mathematicians (the idea of assigning thesis subjects was totally foreign to French professors of that period), but he had no choice: his lower degree was in mathematics and, while Poincaré did little research in probability, he was in charge of teaching it.

Bachelier's tragedy was to be a man of the past and of the future but not of his present. He was a man of the past because his invention, continuous time stochastic processes, was grafted onto the historical roots of probability theory, namely, the study of gambling as implemented by *La Bourse.*

He was a man of the future, both in mathematics (witness the above letter by Lévy) and in economics, where he is acknowledged as the creator of the prob-

abilistic concept of "martingale" (this is the proper formulation of the notion of a *fair game* or of an *efficient market*, see Chapter 37), and he was well ahead of his time in understanding many specific aspects of uncertainty as related to economics.

He owes his greatest fame to the concept that prices follow the Brownian motion process. Unfortunately, no organized scientific community of his time was in a position to understand and welcome him. To gain acceptance for his ideas would have required supreme political skills that he evidently did not possess.

To survive and go on producing new works under these circumstances, Bachelier had to feel strongly about the importance of his work. In particular, he felt that, among his original scholarly contributions, one of the most important was the theory of the diffusion of probability. Here is a quote from an unpublished *Notice* that he wrote in 1921 (while applying for some unspecified academic position). Writing in the third person, he hailed "images taken from natural phenomena, like the theory of radiation of probability, in which &lbt.he&rbt. likens an abstraction to energy—a strange and unexpected linkage and a starting point for great progress. It was with this concept in mind that Henri Poincaré had written, 'Mr. Bachelier has evidenced an original and precise mind.'"

The preceding sentence is taken from the already-mentioned report on the dissertation, which deserves further excerpting: "The manner in which the candidate obtains the law of Gauss is most original, and all the more interesting as the same reasoning might, with a few changes, be extended to the theory of errors. He develops this in a chapter which might at first seem strange, for he titles it 'Radiation of Probability.' In effect, the author resorts to a comparison with the analytical theory of the propagation of heat. A little reflection shows that the analogy is real and the comparison legitimate. Fourier's reasoning is applicable almost without change to this problem, which is so different from that for which it had been created. It is regrettable that &lbt.the

author&rbt. did not develop this part of his thesis further."

Poincaré, therefore, had recognized that Bachelier had advanced to the threshold of a general theory of diffusion. However, Poincaré was notorious for lapses of memory. A few years later, he took an active part in discussions concerning Brownian diffusion, but had forgotten Bachelier's 1900 dissertation.

Other comments in Bachelier's *Notice* are also worth summarizing: "1906: *Théorie des probabilités continues.* This theory has no relation whatsoever with the theory of geometric probability, whose scope is very limited. This is a science of another level of difficulty and generality than the calculus of probability. Conception, analysis, method, everything in it is new. 1913: *Probabilités cinématiques et dynamiques.* These applications of probability to mechanics are the author's own, absolutely. He took the original idea from no one; no work of the same kind has ever been performed. Conception, method, results, everything is new."

The hapless authors of academic *Notices* are not called upon to be modest, and Louis Bachelier did exaggerate to some extent. Moreover, he gave no evidence of having read anything written in the twentieth century. Unfortunately, his contemporaries discounted everything he said and refused him the position he was seeking!

Poincaré's statements are paraphrased, with permission, from a report filed in the Archives of the Pierre and Marie Curie University (Paris VI), heir to the archives of the former Faculty of Sciences of Paris. This fascinating document is written in the lucid style characteristic of this author's famed popular writings. Someone should publish more extensive selections from Poincaré's letters and confidential reports to universities and academies. Poincaré deserves to be made available. As of today, a whole aspect of his personality is absent from his books and his *Collected Works.*

Does anyone have further knowledge of Bachelier's life and personality?