Discovery of Cosmic Fractals
by Yurij Baryshev & Pekka Teerikorpi.
Singapore: World Scientific, 2002.

Foreword by Benoit Mandelbrot

A contribution to the history of the conditional cosmological principle

The authors have asked me to present to the reader this most attractive and wide-ranging book of theirs. But this book hardly needs being presented! It is so well-informed of the history of its subject that it stands by itself and deserves to be appreciated from at least two viewpoints. Firstly, as the second word in the title suggests, it introduces to a very wide potential readership many facts and theories of cosmology. The style is precise but highly personal, a relief after too many introductions beholden to a stylistic formula. Secondly, as the remainder of the title suggests, this book has the unusual distinction of being a notable contribution to the history of ideas.

The authors also asked me to add to their work by describing the history of the fractal model of the cosmos, as I lived it during a long period when this model was not even criticized but simply dismissed. Specifically, they want to hear the story of how I came to improve upon the work of past giants by arguing that the cosmological principle should be restated in a conditional form. It is a pleasure to oblige.

The contrast between the homogeneous and fractal models of the universe will have to be resolved on its own terms. But conceptually this contrast is not isolated. Quite to the contrary: as I see it today, it is best understood and appreciated as a major facet of a long-standing, though also long-subdued, dialectic opposition between thoughts directed towards smoothness or roughness.

Instead of “roughness”, I used to speak or write of “irregularity”, but I now view that word as not only pedantic but clearly inappropriate. Indeed, it means “non-regularity” and somehow implies that regularity came first and roughness only later. In historical fact, the precise opposite was the case. Indeed, before the emergence of science and engineering, nearly-plane surfaces used to be exceptional in Man's experience, a rare example being a quiet body of water and an even rarer one being a crystal. Circles were suggested by the full moon, a pebble's effect on a flat body of water, or an eye's pupil and iris. Spheres were suggested by some seeds.

Is it despite or because of their being exceptional, that those examples proved so extraordinarily attractive (or repellent?) to Mankind? Indeed, many cultures took idealized forms of smooth shapes as foundations of more or less developed but, in effect, universal forms of pre-classical geometry. Classical geometry followed when Euclid collected and organized all that was known and went far beyond by introducing the axiomatic method. That geometry has been developing ever since. It is not only the foundation of the overwhelming bulk of the study of nature, but also of the study of many aspects of culture — a short word I like to use to denote all of Man's works. The old technology — as exemplified by highways, tables, and knives — had no choice but to tolerate roughness and only then as second best, behind an ideal represented by perfect flatness or smoothness.

For roughness, to the contrary, no comparable theoretical developments can be cited. No notion of “perfect roughness” was defined and made the focus of systematic study.

Does the preceding thumbnail history imply that, among shapes that matter to humanity, the development of technology and science witnessed a thorough “victory” of the flat over the rough? In the 1960s, there was no explicit awareness of past battles between rough “natives” and a smooth “conqueror”. That is, the word “victory” would not have come to mind. Quite appropriately, everyone identified geometry, science, and technology as centered on the flat or smooth, allowing as exceptions the smooth endowed with a few odd corners, or the smooth perturbed by the downy skin of a peach.

However, a development that was destined to become the seed of a major exception was planted around 1900. Within the official history of ideas I learned as a student, this seed-to-be consisted in esoterica that everyone called “mathematical monsters”. They were described as having no past and no precedent, as having been “invented” wholly armed from the brow of Jupiter and specifically intended to have no conceivable use in the sciences. Considerate teachers kept the monsters away from impressionable young minds.

Many of the original monsters happen to be self-similar, that is, made of parts deduced from the whole by a linear reduction. But the development of the mathematical esoterica immediately generalized over this property. Generality is praised by mathematicians for its own sake. Therefore, in order to use those esoterica, the first thing I had to do was to reestablish self-similarity and to lean on it heavily as a principle of invariance. But I am getting ahead of the story.

What follows is necessarily autobiographical. Due to an education which events perturbed to an extreme degree, I combined three features that seemed around 1950 to be mutually exclusive: a close acquaintance with the monsters in question, fluency in probabilistic esoterica, and (more surprisingly) a passionate wish to find some regularity in parts of both nature and culture that science had not previously touched. Those parts of culture go beyond highways, tables and knives and include the financial markets and other large but mostly uncontrolled designs such as the internet. They exemplify a high level of perceived "messiness" that I hoped to tame into mere complexity. Early on, I began to combine freely all those high and low caste concerns together, and in due time I conceived around them a new geometry that I had the privilege to name. In 1975, I coined for it the term, "fractal".

What is fractal geometry and what do I hope for it? Down to earth, it is the first organized step towards something that did not exist: a theory of roughness that could, to some extent, complement the great and diversified theory of smoothness.

How does this ambitious program concern clustering, therefore affect cosmology? The path that led me to fractal geometry began in a context altogether different from cosmology, but one that, in due time, made galaxy clusters come to mind unavoidably. The first step was taken in the early 1960s when I studied the clustering of errors in telephone channels and (metaphorically) found that what seemed like a small nut could only be opened by an intellectual sledge-hammer. That is, I had to devise tools that seemed unnecessarily powerful. Then, in the mid 1960s, chance reading made me turn to clustering of matter in the universe, and those unnecessarily powerful tools became handy and suggested the now-familiar "conditional" form of the classical "cosmological principle". Let me retell those events more slowly.

To describe the background, a model of galaxy clustering had been proposed in the 1950s by Jerzy Neyman and Elizabeth Scott. They postulated a compound Poisson process, constructing deliberately a randomized form of hierarchy. Since the Poisson process of constant density yields only a shadow of clustering, they took it as a first approximation and proceeded to improve it recursively, as follows. First, they injected clusters by allowing the Poisson density to vary according to a master process. Next, they injected superclusters by varying the master process density according to a supermaster process. The Neyman-Scott procedure could be extended as far as fancy wished, but it was a truly "Ptolemaic" throwback that had few admirers. Nearly everyone I respected dismissed it as an arbitrary exercise in curve-fitting. It was true that many desired features were present, but only for the reason that they had been very deliberately put in. That is, the model was far from being parsimonious. Nevertheless, Neyman enjoyed such great authority that in 1962 the engineers concerned with clusters of errors on telephone channels invoked the same Ptolemaic compound Poisson process.

The very different tack I took began with a bit of folklore. The engineers with whom I was working told me that, somehow, error clustering was the same at all scales. For example, subdivide a sample of duration T into equal subsamples and "mark" all the subsamples that include at least one error. Folklore asserted that the marked subsamples follow the same cluster pattern irrespectively of the value of T. This represented a property of invariance by reduction or dilation that, soon afterwards, I called self-similarity.

Ten years later, self-similarity also entered statistical physics as being a form of exact renormalizability. But — notwithstanding recently coined anecdotes to the contrary — statistical physics had no influence whatsoever on my scaling/fractal approach to clustering and conditional cosmographic principle.

Back to the story. I knew a restricted and limited form of clustering that is designed into a "monster" set defined in 1883 by Cantor (to run ahead of the story, the 1907 model of galactic clustering of Fournier d'Albe was — consciously or not — a three-dimensional Cantor set!) Unfortunately, practically every physicist had accepted the mathematicians' proclamations since 1900 and believed that the Cantor set could have no role in the modeling of nature. Moreover, there was no point of fighting those prejudices because it was true that the Cantor set could serve only if modified very deeply. Its hierarchy is relentless and deliberately “designed in”; its self-similarity applies only to certain values of *T* and *n.* Whether planned or not, it is another “Plotemaic” construction carried to the infinitely small and large: epicycles down to zero and “subcycles” forever. Worse, it has a privileged center, a lethal defect that can only be avoided by being replaced by an infinite sequence of increasingly “global” centers that asymptotically oscillates between plus and minus infinity.

My second good fortune was to know that self-similarity could be preserved and the lethal defects of the Cantor set avoided by replacing it by a certain random construction. I later called it a “Lévy dust” and described the method of generating it as a one-directional “Lévy flight” (to run ahead of the story again, a three-dimensional variant generates my fractal model of the “seeded universe” of galaxies.)

Moreover, I had discovered that the Lévy dust is magnificently “creative”. The construction itself involves a single parameter that is not a spatial scale but a fractal dimension. It is absolutely not Ptolemaic, not hierarchical. The true novelty of this work resided in two features.

Firstly, humans invariably perceive a sample of Lévy dust as involving an infinite clustering hierarchy. To restate this, clustering and superclustering are not present due to being willfully injected, but because they follow necessarily from the basic scaling invariance, that is, from fractality.

Furthermore, the Lévy dust has no privileged center. More precisely, every center Ω that belongs to the Lévy dust *L* yields exactly the same statistical distribution for the other points in *L.* But a point Ω chosen at random is very different: it falls (with probability 1) into a great void; that is, one whose duration is (with probability 1) infinite. In probabilistic terms, my model required serious reexamination of essential concepts: it was not stationary, but only satisfies a weaker property I called conditional stationarity.

A brouhaha ensued. In my model the probability of errors is zero, but engineers know that the actual probability may be very small but is surely not zero. Hence, my model was bound to break down for large enough error-free time intervals, and results in a well-defined unconditioned distribution being available for every origin 0. I was told that this made it unnecessary to condition Ω to belong to *L. I* agreed in principle but pointed out that, in practice, the said unconditional distribution is the product of two factors.

The first was a power-law with a solid empirical foundation. The second was a multiplier equal to the probability of an arbitrary Ω falling in *L*, which is at best a highly uncertain quantity. Therefore, it was best to avoid relying on the multiplier and observe that everything of interest reduced to the power-law factor.

By the mid 1960s, the above described structure of clustering on the line was essentially completed. At this point, a 1954 article by G. Gamow in “Scientific American” came to my attention and made me aware of Charlier's publication on the clustering of galaxies. Instantly, my mind extended all that precedes from 1 to 3 dimensions, transposing it into new terms relative to cosmology. I read Charlier and followed up his credit to Fournier d'Albe, who had been thoroughly forgotten.

My work was an essential improvement on those predecessors. Firstly, clustering and the appearance of hierarchy were no longer specifically inserted but followed as necessary consequences of scale invariance, that is, fractality. Secondly, the sledge-hammer of fractality was no longer used to crack a nut but a major issue. Conditional stationarity instantly achieved a greatly increased “status” by being translated into a conditional form of the cosmological principle. Thirdly, the question of why the universe should be expected to be clustered was thoroughly modified and became easier to handle. Once it has been transformed into the question of why it should be fractal, another bit of high mathematical esoterica came back to my mind, namely, the so-called “Frostman's potential theory” that relates fractals to attraction proportional to an inverse power of distance.

Having described how I avoided the deficiencies of the Fournier d'Albe-Charlier approach, I hasten to report how I experienced another great surprise that is of broad character, very different from this book's scientific focus but closely relevant to its historical form. Evidence started accumulating that the study of roughness (more pedantically, of extreme irregularity) was not starting with the proverbial empty table, “tabula rasa”. No study interpretable in terms of roughness had been carried on to a technical level until I “tamed” the “monsters”. However, relevant general thoughts were very much part of the historical record. More generally, my books “Les objets fractals” and “The Fractal Geometry of Nature” were widely read and commented upon, even before the former actually appeared and increasingly so after the publication of the latter. As a result, evidence came forth and continues to accumulate that the mainstream of mathematics, science, and engineering was the only context (if I dare use "only" for such a purpose) in which the flat overwhelmingly prevailed over the rough. The rough that was not relegated to mathematical esoterica had survived in many odd corners all over human experience. As usual, history was written by those who had prevailed, and my teachers' perception of the history of ideas was altogether biased.

This brings us back at long last to the book by Baryshev and Teerikorpi. It is original and personal (a form of praise under my pen) and worthy and useful on its own terms of cosmology. But it is also an important contribution to a much broader task triggered when I conceived and organized fractal geometry, used it in many fields, and only later realized that fractals had a past beyond 50 to 100 years of mathematical esoterica.

Searching and documenting all this past far exceeds any single individual's qualifications, but motivates several recent books. A book by Eglash showed that forms identified after the fact as fractal have long characterized African art and design. A forthcoming book by Jackson will show that proto-fractals also permeate religious iconography in Europe and many parts of Asia. A great contemporary composer, G. Ligeti, pointed out that music also turns out to have fractal aspects. Some old theories had clearly outlined them, but only dimly. Altogether, an intuitive understanding of fractality can be traced to the dawn of humanity.

Let us now return to the place organized sciences take among Man's eternal questions. Most are younger than art, but astronomy/cosmology is arguably an exception. Illustrating quite literally the concept of an endless frontier, it is also both one of the youngest and most active. As such, it is also (most unfortunately) one of the most oblivious of its rich past. It is therefore perfectly proper that the first systematic examination of the prehistory of fractals in a domain of science should concern this field.

Before concluding, one must comment on this admirable scholarship's final goal. Now that science is organized and books are plentiful, each scientist has many teachers and, if successful, many disciples. A subject's history is documented and can be followed step by step, in particular as it migrates from one favorite topic to another. But this book deals with times before science became organized, when scientists were few and isolated from one another in time and space, and interacted little. The situation that used to be the rule had a late but illustrative example in a topic of mathematics that started in 1872 with Weierstrass. (While it does not matter here, let it be said that it concerned "continuous but non-differentiable functions".)

Fifty years later, it was revealed that Weierstrass had been anticipated by fifty years by Bernard Bolzano. However, the latter did not publish and no evidence is known that he exerted an indirect influence.

To make my next point, recall that the French language distinguishes between "grande et petite histoire". In general, "petite histoire" is irrelevant gossip and the like. But in the history of ideas and of sciences it may help make a very important distinction. The "grande" history of that topic in mathematics did not begin until Weierstrass. The work of Bolzano only stars in "petty" history. It is illustrative and extraordinarily attractive, and I take pride in having moved it into wide public awareness. But a distinction must be drawn clearly between the streams of history which reach the "sea" and those which become lost in the sands. How does this distinction relate to this book? Beyond the "protofractal" thoughts of Kant, Charlier, and others already quoted, Baryshev and Teerikorpi have literally unearthed many other authors whose work remained isolated, undeveloped, and with little influence. They belong to the grand history of ideas, but only to the petty history of the fractal model.

The remarkable book I now have the privilege and pleasure of recommending to the reader is a joy to read and taught me a great deal. I developed a strong admiration for the authors' expository skills and I am wowed by the historical and geographical breadth of their scholarship. I wish them all the best.

Benoit Mandelbrot

Sterling Professor of Mathematical Sciences, Yale University, New Haven, USA
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