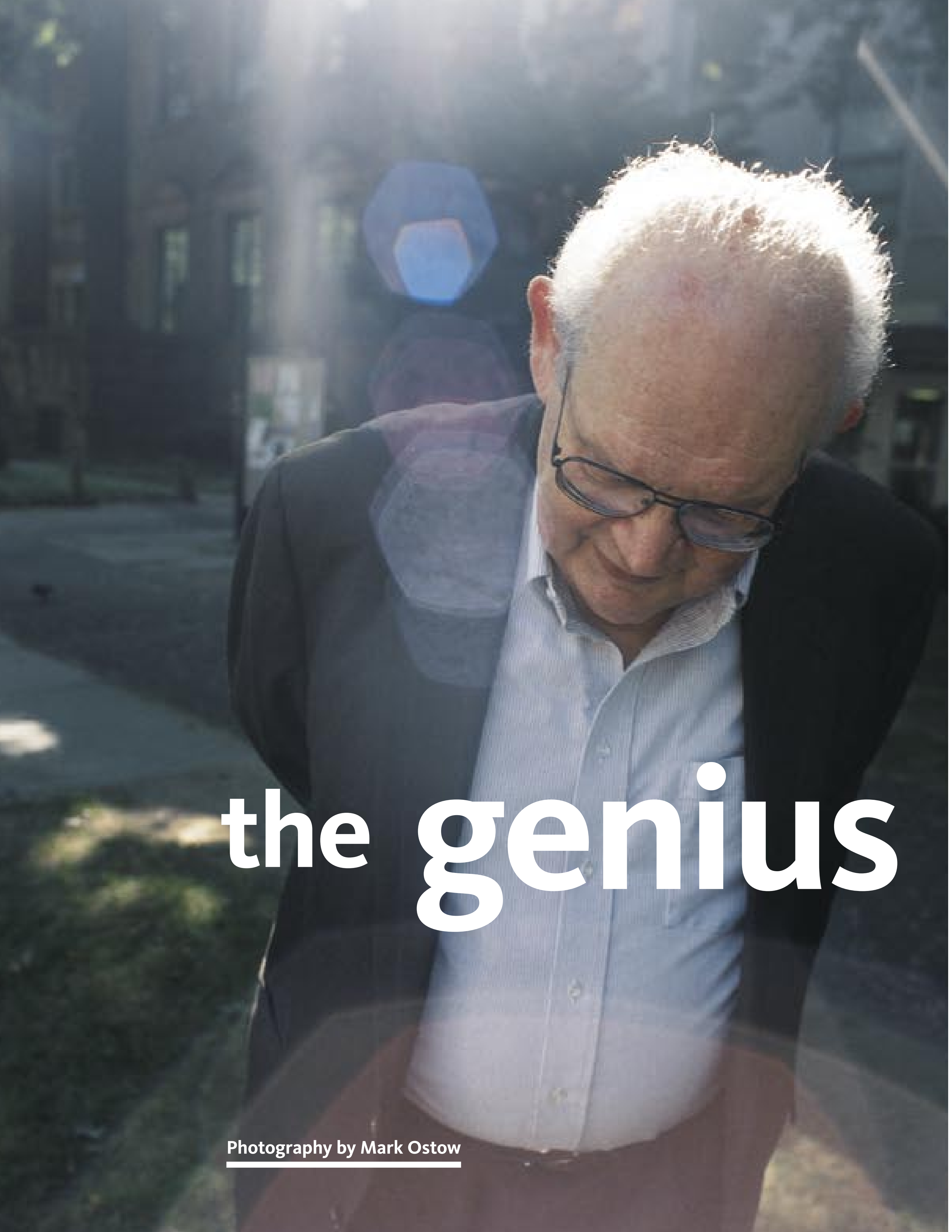


the genius of the unpredictable

The man who invented fractals looks at the stock market, math education, and the wild improbability of his own life.

Steve Olson '78

Photography by Mark Ostow

In a second-floor lecture hall of the Yale math department, in front of about 100 New Haven–area high school teachers, Benoit Mandelbrot is delivering a talk that is as irrepressible as the late-August rainstorm drenching the city outside. Over the course of 45 minutes, he mentions cotton prices, the flow of the Nile River, the Japanese artist Hokusai Katsushika, soap bubbles, the distribution of galaxies in the universe, fractional dimensions, Hollywood films—a dizzying array of ideas, conjectures, and images—all presented in a self-effacing aw-shucks tone that instantly puts the teachers at ease. They would have no way of knowing, based on his manner alone, that they are listening to one of the most famous mathematicians in the world.

Or is Mandelbrot a mathematician? "Sometimes people ask me whether he should be classified as a mathematician, a physicist, an economist, or a hydrologist," says his math department colleague Peter Jones. "My response is, 'Yes.'" Mandelbrot is most famous as the inventor of fractal geometry, which, loosely speaking, is a way of measuring the roughness of an object or phenomenon. But if a mathematician is a device for converting coffee into theorems, as the number theorist Paul Erdős famously claimed, then Mandelbrot is atypical in the extreme. He has proved very few theorems during his six-decade career; rather, his renown rests largely on his ability to observe, describe, and generalize the shapes he has encountered in nature. Before coming to Yale in 1987, he worked for 30 years, not in academia, but at IBM. He has always been the consummate outsider, rejecting the strictures of traditional academic disciplines. "The irony," says Jones, "is that he has been right so many times he may end up an insider."

Mandelbrot's influence is pervasive in mathematics, science, and technology. A website that accompanies Yale's undergraduate math course on fractals (http://classes.yale.edu/fractals/welcome.html) lists 95 examples of fractal phenomena in society and nature—from "African art" to "Charles Wuorinen," a twentieth-century American composer who has drawn on fractals in his work. Last year the Japanese emperor honored Mandelbrot with the Japan Prize for "outstanding achievements that contribute to the progress of science and technology and the promotion of peace and prosperity." His most recent book, *The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward*, returns to one of the subjects that first introduced him to fractals—the wild gyrations of stock market prices. "A lot of his mathematics has been absorbed into the toolkit of people who say that they aren't studying fractals at all," says mathematician and educator Lynn Arthur Steen of St. Olaf College. "It has become part of the culture."

Mandelbrot is turning 80 this November. The years have creased his skin and whitened his hair, but he has the energy, enthusiasm, and curiosity of a college sophomore. In conversation, one topic suggests another, which suggests another, until he's miles from where he started. He still speaks with a hard-to-place European accent, the product of a childhood spent amidst the tumult of World War II. After 17 years of commuting between his Scarsdale home and New Haven, his view of Yale is both affectionate and wry: "The math department here is a remarkably collegial place," he says, "but most of Yale does not even know the math department exists."

He is writing his memoirs—a three-inch stack of manuscript sits in the center of his desk. The many hours of reflection have impressed upon him the wild improbability of his life. "I chose a very difficult path," he says. "It's amazing that it worked. Because I went to someone's office and the blackboard had not been cleaned, I saw a drawing, and that drawing ended up occupying half my life. Had I come ten minutes later and the fellow had cleaned his blackboard, who knows."

"I could see in perfect three-dimensional vision — lines, planes, complicated shapes," he says. "I was cheating, but my strange performance never broke any written rule."

MANDELBROT'S LIFE WAS SHAPED, he says, by "the disasters of this century." He was born in Poland in 1924 into an extended family of physicians, businessmen, and scholars. His mother was a doctor, and she decided not to send her son to first and second grade because she was afraid that he would contract one of the diseases then prevalent in the schools. Instead, he was tutored by a relative—he remembers spending most of his time playing chess, reading maps, and "learning how to open my eyes to everything around me."

As war loomed over Europe, the Mandelbrots realized that the lives they had known could not continue. Earlier, the youngest brother of Mandelbrot's father had emigrated to Paris, where he had risen quickly to a position of prominence in French mathematics. In 1936 the Mandelbrots followed him, moving to Paris and settling in a slum of Belleville.

Mandelbrot has always had an insatiable curiosity and a superb memory. At one point his father bought a set of encyclopedias from a street vendor and Mandelbrot read them all from front to back. But his schooling was spotty—when he arrived in Paris he knew no French, and in school he was two years older than his classmates. Then the war began and education became secondary to survival. Mandelbrot groomed horses and worked in a machine shop ("to this day I have an extremely steady hand," he says, extending his arm to demonstrate). Once he was almost arrested by the Vichy police because he was wearing an overcoat similar to one worn by a Resistance fighter fleeing the scene of an attack; Mandelbrot is sure that he narrowly avoided execution or deportation.

Mandelbrot's family had moved to Tulle, a small town in central France, and then he moved east to Lyon. There, in his nineteenth year, he made a marvelous discovery. He began studying for the dreaded examinations that French students must take to gain entry to elite French universities. Mathematics is emphasized in the exams, but Mandelbrot had never given math more attention than any other subject, other than occasionally discussing the subject with his mathematician uncle. In Lyon, Mandelbrot realized that he could do something that none of his classmates could do. When presented with an equation to solve, he converted it instantly into a geometric shape, manipulated that shape in his mind, and simply read the answer. "I could see in perfect three-dimensional vision—lines, planes, complicated shapes," he says. "I was cheating, of course, but my strange performance never broke any written rule." On the college entrance exams, he received the highest mathematics score in all of France.

And so began his first rebellion. Based on his exam performance, he gained admission to the École Normale Supérieure, the most prestigious college in France, which admitted only 15 science students that year and has a particularly strong program in abstract mathematics and physics. He attended for two days—and then dropped out. He enrolled instead in the École Polytechnique—also a prestigious school, but one that emphasized the applications of mathematics and science more than theory.

His departure from the École Normale was then—and still is—considered a scandal. In a Festschrift honoring his seventieth birthday, the director of the school sentenced him to remain an "honorary freshman for life." He still seems to be mulling over his reasons for leaving. For one thing, his father, who supported the family by manufacturing and selling clothes, favored the Polytechnique. Mandelbrot père thought that a man should have a steady income independent of the whim of the state, which was not the case for many of the scholars produced by the École Normale.

But Mandelbrot's own convictions were the more important factor. Through conversations with his uncle, he knew that French mathematics was being taken over by a group named Bourbaki, which emphasized the purity, rigor, and abstraction of math. This dogma was anathema to Mandelbrot. He thought of himself as a geometer, as someone who drew inspiration from the real world, not from the pristine abstraction of equations. He knew that his mathematical gift would be squelched by Bourbaki.

Besides, he says, he has always had a problem with authority. "I turned 20 at a time when many people claimed to have solved all the problems of everything. Marx claimed to be the Newton of economics, but he was not. You could not draw from Keynes or Freud any kinds of predictions that you could verify. I was very against the idea that so many people wanted to explain things that they couldn't describe or understand.

"The one role model I had for a long time was Kepler. What Kepler achieved was to describe the planets' motions around the sun by ellipses. Ellipses had been devised for no useful purpose, and I felt that their true nature was revealed by this application. My uncle told me that it was too late, because Kepler had lived and died and now one must study some established question. But I had just the opposite taste. I wanted to find a degree of order in some area—significant or not—where everyone else saw a lawless mess. My dream was, in effect, to become the Kepler of complexity."

MATHEMATICIANS ARE OBSESSED WITH AGE. Many think, despite plentiful evidence to the contrary, that mathematicians do their best work when very young and then run out of inspiration. One of the things that may have triggered Princeton mathematician John Nash's schizophrenia, as depicted in both the written and cinematic versions of *A Beautiful Mind*, was his fear, at age 30, that his best work was behind him. The Fields Medal, which is the equivalent in mathematics of the Nobel Prize, is given only to mathematicians age 40 and younger.

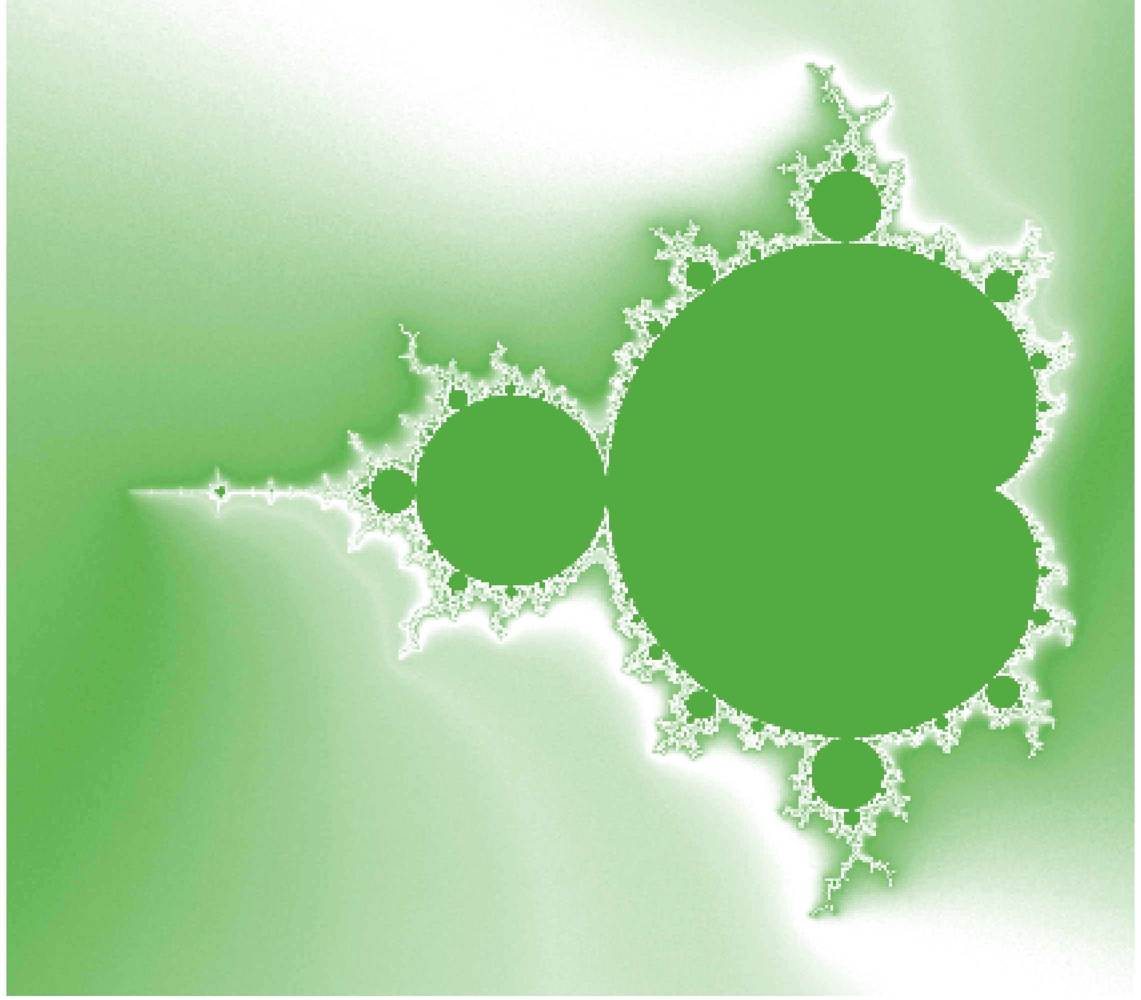
Mandelbrot scorns this obsession. Defying convention takes time. Being an iconoclast is incompatible with "the worship of youth," he says.

Mandelbrot took his time. After graduating from the Polytechnique, he did postgraduate work at the California Institute of Technology, where he studied the turbulence created by jets passing through the atmosphere and the statistical laws that govern the distribution of heat. Later he did postgraduate work at the Institute for Advanced Study in Princeton, under John von Neumann, a key figure in the development of the electronic computer. Von Neumann was worried about Mandelbrot—at one point he asked a mutual friend to keep an eye on the young mathematician because Mandelbrot seemed to be following an intellectually "dangerous" path.

In between his two graduate appointments, Mandelbrot completed a PhD in France. He now seems almost apologetic about that work. The first half of his dissertation, a mathematical treatment of the frequency of word usage, concerned what he calls "a subject that didn't yet exist." The second half, a generalization of the first half using statistical thermodynamics, dealt with a subject "that was viewed as no longer a part of active physics."

Yet Mandelbrot's PhD thesis had a pivotal effect on his career, because it introduced him to the work of George Kingsley Zipf. An independently wealthy scholar who taught at Harvard, Zipf decided in the 1940s that the secret of the world resided in a set of mathematical relationships known as power laws. These laws relate the size of an event to how often that event occurs. For example, another of Mandelbrot's early interests was the distribution of

STEVE OLSON was a National Book Award finalist in 2002. His most recent book is *Count Down: Six Kids Vie for Glory in the World's Toughest Math Competition*.


A.PASIEKA/PHOTO RESEARCHERS, INC.

A graphic representation of the **Mandelbrot set**, a nonlinear equation that generates an infinitely intricate fractal pattern. The image of the Mandelbrot set has become an icon of the twentieth century.

wealth in society. It was known that income distributions follow a power law, with low incomes and high incomes related in a specific way. Furthermore, a power law distribution has a special characteristic. Each part of the distribution reflects the whole. Thus, the pattern of relative income distribution is the same in the top half as the pattern in the top quarter of the distribution, which is the same as in the top tenth of the distribution, and so on.

This property led Mandelbrot to the defining characteristic of fractals. They are geometric patterns whose properties repeat on different scales or with subtle variations. Consider the distribution of galaxies in the universe. When the Hubble space telescope peered at a tiny speck of sky for a solid week, it revealed a fantastic menagerie of galaxies stretching away from us into the black infinity of time and space. Mandelbrot's studies of galaxies' locations had revealed that they follow a fractal distribution, as the Hubble image demonstrated. Individual galaxies form clumps, and the clumps form bigger clumps, and so on. Each larger structure resembles the smaller one, and this repetitive pattern can be described by a power law.

After his postdoc at the Institute for Advanced Study, Mandelbrot served with the French Air Force, then lectured at the University of Geneva, then with the University of Lille, but nothing fit. His papers were widely read, but even to him they seemed scattered, incoherent. Halfway through writing one paper he often was irresistibly drawn to a completely different subject that suddenly attracted his interest.

In 1958 he received an offer to come to the United States to do research for a summer at IBM. Spurred by the rise of electronic computers, IBM was establishing a research division on the Hudson River in New York, which would include a few independent scholars doing unconventional work. "We wanted a free-wheeling component, along with more focused research," says Ralph Gomory, who is now president of the Sloan Foundation and was then Mandelbrot's boss. It was the perfect match. Mandelbrot's summer at IBM turned into a year, then into two years, then three. "I never thought in terms of 35 years at IBM," Mandelbrot says now. "My wife has said that if she'd known we were coming for good, she might never have come."

At first Mandelbrot worked on various computer applications. But he also responded to requests by managers and his own curiosity. A very early interest, partly sparked by an IBM executive, was stock market prices. He began to load economic data into computers so that he could analyze long-term trends. In 1961 he was invited to Harvard to give a talk on his work on the distribution of income. When he arrived at the office of his host, economics professor Hendrik Houthakker, a drawing on the blackboard caught his eye. The drawing had the same shape as one representing the distribution of incomes he was about to discuss in his lecture. Was Houthakker also studying income distributions? No, Houthakker told him, the data showed the distribution of cotton prices over the past century.

That diagram was a revelation for Mandelbrot. Why should cotton prices resemble the distribution of incomes in society? The two seemed to have nothing in common. But the correspondence had to be more than a coincidence. He started looking elsewhere for the shape he had seen on the blackboard, and soon he began to find it—in the flow of rivers, signal errors in transmission lines, turbulence in fluids, earthquake magnitudes, oil field reserves. Each was characterized by a motif repeated with variations, and each could be described by its own power law.

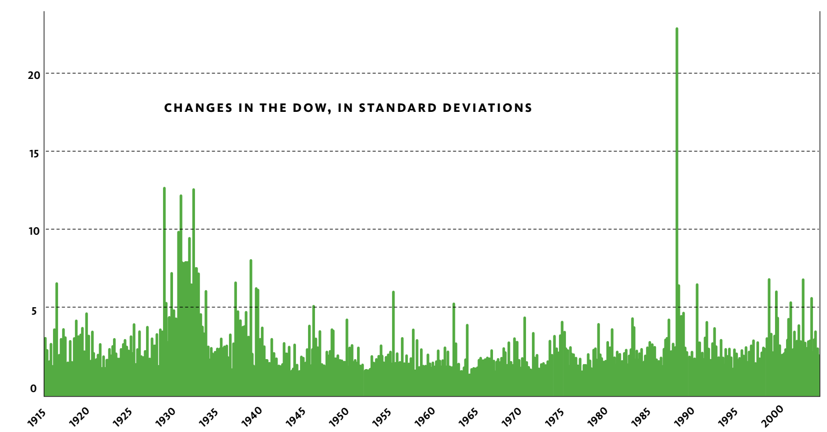
During the 1960s, the separate strands of Mandelbrot's career began to multiply and tie themselves together. He found that repeating patterns and power laws characterized many other features of the natural and social worlds. Moreover, these patterns, despite their apparent complexity, often could be generated by the repeated application of very simple rules. In 1972 he published a manifesto in French that laid out a program for studying these patterns. And in 1975 he gave them a name: "fractals," from the Latin *fractus* for irregular and broken up. The word is now in every dictionary.

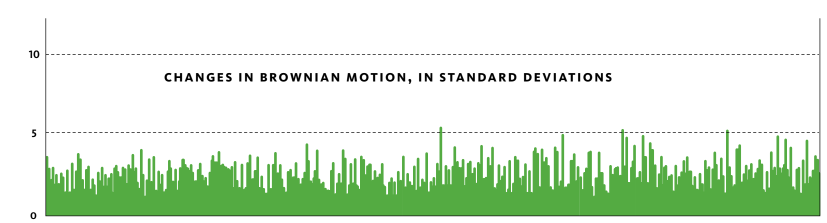
IN THE 1980S THE STUDY OF FRACTALS took off. At the beginning of that decade, Mandelbrot discovered a particular kind of fractal structure that is generated by solving a simple nonlinear equation. Reproduced on coffee mugs, posters, and tee shirts, what is now known as the Mandelbrot set became an icon of the late twentieth century. Meanwhile, other researchers began to realize that fractals could describe phenomena in their own fields—the brain waves observed during epileptic seizures, for example, or the cracks that form in metals.

Fractals soon began to find a wealth of applications. One early center of activity was Hollywood. Many features of the natural world—clouds, mountains, coastlines, vegetation—have fractal shapes. When television and movie directors wanted to produce realistic landscapes without taking their cameras outdoors, they turned to computer artists skilled in the manipulation of fractal images.

Fractals also gained a boost because they are closely related to another hot field of the 1980s—chaos theory. Many aspects of the natural world are inherently unpredictable because of what became known as the butterfly effect: minuscule changes in the input to a system can create immense differences in the output. Fractal phenomena often exhibit this same chaotic behavior.

Mandelbrot has continued to study one of those chaotic phenomena, and it forms the subject of his most recent book. In *The (Mis)Behavior of Markets*, he and coauthor Richard L. Hudson, a former editor at the *Wall Street Journal*, write, "This book will not make you rich." The intent of the book, say Mandelbrot and Hudson, is to keep readers from becoming poor.





The **top graph** above, from *The (Mis)Behavior of Markets*, shows how violently the stock market can behave. The graph tracks every daily change in the Dow Jones index in terms of standard deviations beyond the average change—in other words, it measures how unusual every change is. The short lines are ordinary, small movements of the Dow; the tall lines are large movements. What the chart reveals is the wildly unpredictable nature of the market. The **bottom graph** applies the same technique to Brownian motion, the natural random movement of small particles (such as pollen grains in warm water). The comforting idea that financial markets behave in a Brownian way "underpins almost every tool of modern finance," say Mandelbrot and co-author Richard L. Hudson—but is "clearly contradicted by the facts."

The book's underlying message is that markets are not just unpredictable but wildly unpredictable. Conventional economic theory recognizes an element of randomness in the movement of prices, but it radically underestimates how far and how fast prices can change. "Is the probability of ruin one millionth or one tenth?" Mandelbrot asks. "Everything you see is based on the probability of ruin being negligible, but that's wrong."

The fractal analysis of markets presented by Mandelbrot and Hudson should give investors pause. Large changes in a market, either up or down, are much more likely than students are taught in business schools. In the 1980s, for example, 40 percent of the positive returns from the Standard & Poor's 500 index came during ten days—about 0.5 percent of the total time. Savvy investors know in their gut that markets are a roller coaster, and some are even able to take advantage of sudden price changes. But because of the fractal nature of those changes, market prices are virtually impossible to predict. It's a simple observation—so simple that some argue over whether Mandelbrot was the first to make it. But Mandelbrot certainly was the first to see in market prices the workings of a system so complex that its behavior can be described only by the patterned unpredictability of fractals.

"There is an unkind aspect to that message," he says. "The whole world of economics is enormously more complex than the world of physics. And therefore the teaching of business schools, including Yale's, is unrealistic. Even though economics is a very old subject, it has not truly come to grips with the main difficulty, which is the inordinate practical importance of a few extreme events."

MANDELBROT WAS A LATECOMER to the satisfactions of education. Ensconced at IBM, he had few opportunities to interact with students—until Yale offered him a faculty position.

Mandelbrot often had been a visiting professor at universities, but his experiences had been unsatisfactory. Partly he chafed at the rigidity of traditional math departments, which tend to be vitally concerned with disciplinary borders and with who is doing what. Mandelbrot has rarely seen a border that he didn't want to transgress.

The Yale math department was different. It had a reputation for hiring excellent mathematicians (members of the department have received many awards, including the Fields Medal), and then letting them follow their instincts. "I never expected it would be such a good arrangement, " Mandelbrot says. And the department went out of its way to make him feel at home. So that he could become a Sterling Professor, in 1999, Yale tenured him at age 75; Mandelbrot is the oldest professor ever to receive tenure at Yale.

Among students, the Yale math department has rather a different reputation. They know it primarily through its introductory courses, especially calculus. The combination of a tough subject with less-than-stellar instructors often has left Yale's calculus courses near the bottom in student evaluations.

Mandelbrot knew a professor named Michael Frame at Union College in New York who had been a leader in reforming math education there. Together with allies in the math department, Mandelbrot convinced the Yale administration to hire Frame as an adjunct. He arrived in 1997 and is reorganizing Yale's calculus instruction by using computers and the web to illustrate fundamental principles. "Michael has arranged it very well," says department chair Andrew Casson. "The software is now sufficiently simple to be used on a daily basis."

Frame and Mandelbrot have undertaken two other joint ventures. They have organized a two-week end-of-summer workshop for local college and high school teachers on how to use fractals in the classroom. It's a way to give something back to New Haven, Mandelbrot says, and the teachers who attend the workshop each year are knowledgeable and enthusiastic.

Mandelbrot and Frame also have created one of the most popular courses in the history of the department. Math 190, "Fractal Geometry," attracts about a hundred students each year through its clever combination of computer simulations, engaging mathematics, and real-world applications. Fractals are "a very effective language for describing much of nature, and also some kinds of art, music, and literature," says Frame. "Students are intrigued at finding a new kind of pattern, one they have seen all their lives but have failed to recognize."

Nothing seems to please Mandelbrot more than the success of Math 190. From his childhood, Mandelbrot has been a student of the natural world. Now, he is teaching others how to see things that he was the first to see. "There is something fundamentally attractive to the human mind about fractals," he says. "Once fractal shapes became known, people viewed them as beautiful. But the shapes were always there. People just hadn't looked at them carefully." Y