

Fractals: An Animated Discussion with Edward Lorenz and Benoit Mandelbrot.

A film by H.-O. Peitgen, H. Jiirgens, D. Saupe, and C. Zahiten. W H. Freeman and Co., New York, 1990. VHS/Color/63 min. \$59.95 for individuals; \$149.95 for educational purposes. ISBN 0-7167-2213-5.

Warning: If you are a mathematician, you are not allowed to read this article. Like the videotape under review, these remarks will be addressed to the general public, the countless thousands of nonmathematically trained readers of SIAM Review.

So you're not a mathematician, but you've heard about chaos and fractals and you want to learn more. What will you see in this video? Well, you will see a lot of new and interesting mathematical ideas from the fields of mathematics known as fractal geometry and dynamical systems. You will undoubtedly come away with an appreciation for the great beauty of mathematics. You will meet two of the most engaging personalities in the field, Ed Lorenz and Benoit Mandelbrot. You probably will not understand everything you see, but that's OK. Watch the video again. You will get more out of each successive viewing. If you are worried that all those formulas and abstractions that drove you away from mathematics in the first place will come back to haunt you again, you can breathe a sigh of relief—there is not a single formula in the entire video! (That's why I forbade mathematicians to read this review. They would probably gasp in horror: Newton's method with no formulas? The Mandelbrot Set without mentioning $z^2 + c$? Heresy!)

In any event, you will definitely come away with an intuitive grasp of the two central themes of the video, self-similarity and sensitive dependence on initial conditions. These two important ideas are beautifully described by means of a metallic pendulum suspended over three magnets painted red, blue, and yellow. Let the pendulum swing and the resulting motion is quite chaotic. Change the initial position of the pendulum and you observe startlingly different behavior. After racing

around in a complicated pattern for a while, the pendulum eventually settles down on one of the three magnets. If you paint the picture of the initial conditions that lead to attraction to the red, blue, or yellow magnets, you find an amazingly complicated picture. These "basins of attraction" are intertwined in a manner that cannot be resolved by magnification of the picture. This is self-similarity. Furthermore, slight changes in where the pendulum begins its motion often leads to very different fates for the resulting motion. This is sensitive dependence on initial conditions. So, in this one easily understood example, we meet both fractals and chaos in a highly visual and accessible format.

In this video you will see other examples of these phenomena, including Newton's method for finding the roots of a function. You may have a little trouble understanding what the planar pictures really mean, and you certainly won't understand why these images change as parameters are adjusted. But that's not the point: the goal here is to show you that very different phenomena produce images of incredible complexity and that self-similarity and chaotic behavior are two concepts that are often linked.

One very nice feature of the video is the fact that (despite the lack of formulas) it's the mathematics that is center-stage here. There are no artificial claims of great applicability to other areas of science and engineering, no promises of great new scientific developments resulting from fractals, just an honest attempt to share some of the delightful mathematics with nonmathematicians.

There are some flaws in the video. The producers occasionally cannot control themselves and go overboard with fancy graphics. Thus the Mandelbrot Set, painstakingly defined as a subset of the plane, suddenly looms as a three-dimensional object without any explanation. The Julia set for Newton's method, again a subset of the plane, eventually ends up on the sphere with other planetary Julia sets orbiting around it.

There is also the inevitable hype that seems to swirl around fractal geometry. The video jacket proclaims that "Fractal geometry is perhaps the most exciting discovery of contemporary mathematics." If you want to see your favorite mathematician bristle, just say those words to him or her.

The commentator contributes similar overstatements about chaos. For instance: "Scientists were intoxicated by the discovery that chaos is not an exception—overlooked in past centuries" and "scientists stood speechless before a phenomenon present in even simple systems." Intoxicated? Speechless? I don't think so.

Also, as the jacket proclaims, the video features "music composed according to fractal principles." In my experience, whenever you mix music and fractals, the result is total disaster, both scientifically and musically, and the current tape is no exception. You will be tempted to turn the soundtrack off, but you can't, because then you'll miss the wonderful interviews with Benoit Mandelbrot and Edward Lorenz. These interviews are the highlight of the tape. I found the discussions of both men's early mathematical backgrounds most interesting. Lorenz talks about his days as a mathematics graduate student under G. D. Birkhoff and Mandelbrot recalls both his debts to and mathematical quarrels with his uncle, the esteemed complex analyst, Szolem Mandelbrot. Mandelbrot, as always, raises a few hackles with his jabs at contemporary mathematics and its ingrown culture. He seems particularly outraged at the "algebraicization" of modern mathematics, at the expense of his own passion, geometry. He recounts how his own uncle told him that "geometry is something you must outlive ... it is only for children." These reminiscences certainly help to show the personal side of mathematics and that research mathematicians are (surprise, surprise!) human too.

Finally, you must realize that there are the inevitable omissions in the presentation. We meet (briefly) the forefathers of chaos and fractals, Cantor, Hausdorff, and Julia. But there is no mention whatsoever of the pioneering work of Poincaré, and Smale, who is the modern father of chaos, also receives nary a mention.

But I dwell on the negatives. Any nonmathematician will love this video. It is done with superb attention to detail, especially the computer animations. You very definitely get the sense that mathematics is an alive and beautiful discipline, and for this I congratulate the producers.

Addendum: For the research mathematicians who so shamelessly read the above, I have one final comment. This tape will undeniably make you wince once or twice. I

suggest you watch it only when accompanied by a group of students. The idea of a warning label such as on cigarette packages or R-rated movies is appealing. Warning: mathematicians viewing this film must be accompanied by a person under the age of 17. I suspect that students, especially high schoolers or liberal arts majors, will have a vastly different impression of this video than research mathematicians. Showing it in a class comprised of such students would be a wonderful idea. The video has a lot to say about mathematics, and it does so in a wonderfully non-intimidating manner. As Mandelbrot says in the video: "Fractal geometry gives us opportunity for getting more people interested in mathematics a little bit, some a little bit more, some even further, and then to have a new and more varied mathematical community is an extraordinary opportunity ... [chaos and fractals provide] a great opportunity for mathematicians to help end their isolation from the rest of activities of man and to spread the **awareness of the beauty of mathematics.**"

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