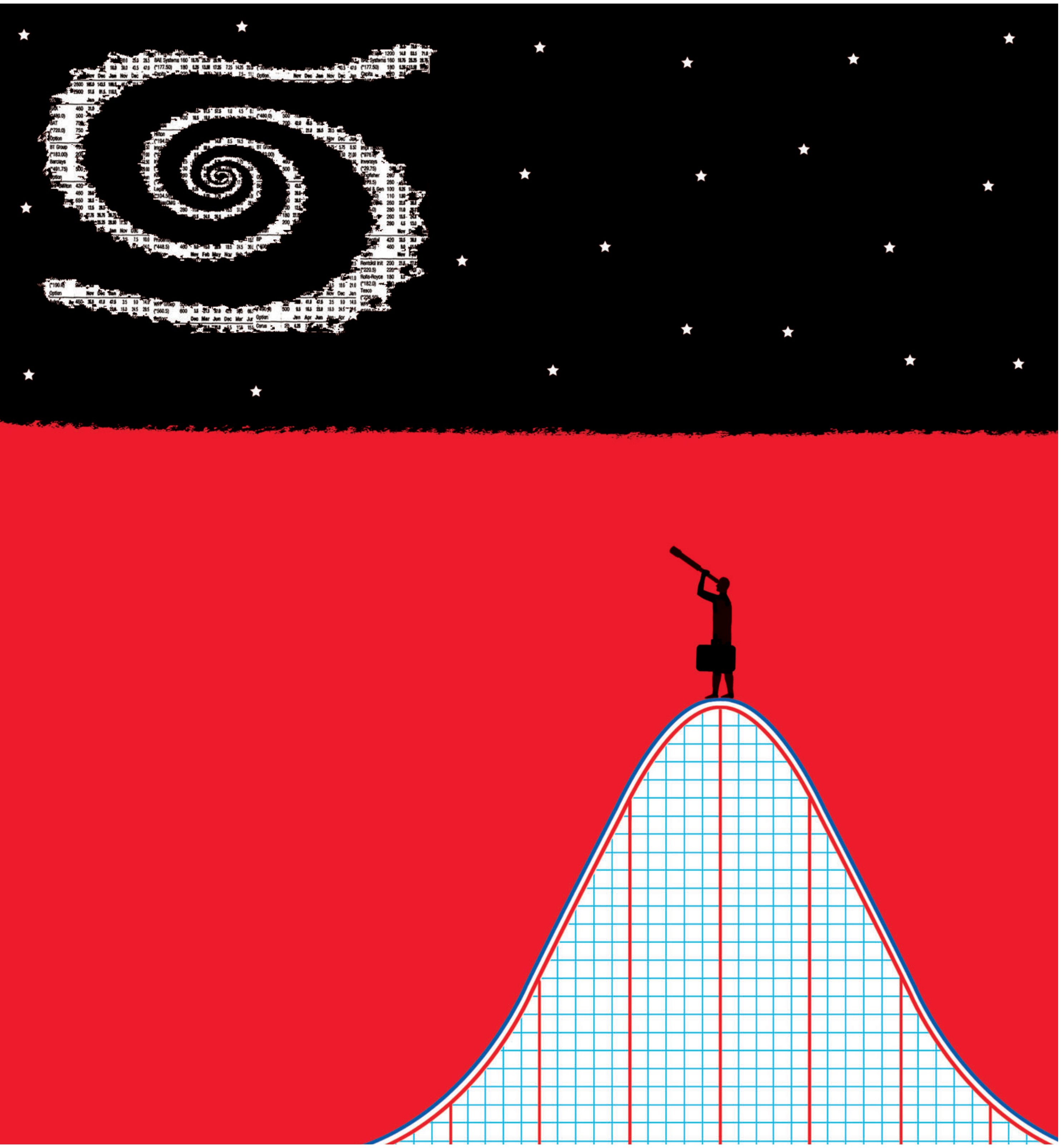

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The Gaussian and fractal models: observations and consequences

1 By itself, no single number can characterise uncertainty and risk but, as we have seen, we can still have a handle on it so long as we can have a table, a chart and an open mind.

2 In the Gaussian world, standard tables show that 67 per cent of the observations fall between -1 and +1 sigma. Outside of this, sigma loses its significance. With a scalable distribution, you may have 80 per cent, 90 per cent, even 99.99 per cent of observations falling between -1 and +1 sigmas. In fractals, the standard deviation is never a "typical" value and may even be infinite!

3 When assessing the effectiveness of a given financial, economic or social strategy, the observation window needs to be large enough to include substantial deviations, so one must base strategies on a long time frame.

4 You are far less diversified than you assume. Because the market returns in the very long run will be dominated by a small number of investments, you need to mitigate the risk of missing these by investing as broadly as possible. Very broad passive indexing is far more effective than active selection.

5 Projections of deficits, performance and interest rates are marred with extraordinarily large errors. In many budget calculations, US interest rates were projected to be 5 per cent for 2001 (not 1 per cent); oil prices were projected to be close to $22 a barrel for 2006 (not $62). Like prices, forecast errors follow a fractal distribution.

6 Option pricing models, such as Black-Scholes-Merton, are strongly grounded in the bell curve in their representation of risk. The Black-Scholes-Merton equation bases itself on the possibility of eliminating an option's risk through continuous dynamic hedging, a procedure incompatible with fractal discontinuities.

7 Some classes of investments with explosive upside, such as venture capital, need to be favoured over those that do not have such potential. Technology investments get bad press; priced appropriately (in the initial stages) they can deliver huge potential profits, thanks to the small, but significant, possibility of a massive windfall.

8 Large moves beget large moves; markets keep in memory the volatility of past deviations. A subtle concept, fractal memory provides an intrinsic way of modelling both the clustering of large events and the phenomenon of regime switching, which refers to phases when markets move from low to high volatility.

of variables such as financial markets. For example, a level desribed as a 22 sigma has been exceeded with the stock market crashes of 1987 and the interest rate moves of 1992.

The key here is to note how the frequencies in the preceding list drop very rapidly, in an accelerating way. The ratio is not invariant with respect to scale.

Let us now look more closely at a fractal, or scalable, distribution using the example of wealth. We find that the odds of encountering a millionaire in Europe are as follows:

Richer than 1 million: 1 in 62.5
Richer than 2 million: 1 in 250
Richer than 4 million: 1 in 1,000
Richer than 8 million: 1 in 4,000
Richer than 16 million: 1 in 16,000
Richer than 32 million: 1 in 64,000
Richer than 320 million: 1 in 6,400,000

This is simply a fractal law with a "tail exponent", or "alpha", of two, which means that when the number is doubled, the incidence goes down by the square of that number – in this case four. If you look at the ratio of the moves, you will notice that this ratio is invariant with respect to scale.

If the "alpha" were one, the incidence would decline by half when the number is doubled. This would produce a "flatter" distribution (fatter tails), whereby a greater contribution to the total comes from the low probability events.

Richer than 1 million: 1 in 62.5
Richer than 2 million: 1 in 125
Richer than 4 million: 1 in 250
Richer than 8 million: 1 in 500
Richer than 16 million: 1 in 1,000

We have used the example of wealth here, but the same "fractal" scale can be used for stock market returns and many other variables. Indeed, this fractal approach can prove to be an extremely robust method to identify a portfolio's vulnerability to severe risks. Traditional "stress testing" is usually done by selecting an arbitrary number of "worst-case scenarios" from past data. It assumes that whenever one has seen in the past a large move of, say, 10 per cent, one can conclude that a fluctuation of this magnitude would be the worst one can expect for the future. This method forgets that crashes happen without antecedents. Before the crash of 1987, stress testing would not have allowed for a 22 per cent move.

Using a fractal method, it is easy to extrapolate multiple projected scenarios. If your worst-case scenario from the past data was, say, a move of –5 per cent and, if you assume that it happens once every two years, then, with an "alpha" of two, you can consider that a –10 per cent move happens every eight years and add such a possibility to your simulation. Using this model, a –15 per cent move would happen every 16 years, and so forth. This will give you a much clearer idea of your risks by expressing them as a series of possibilities.

You can also change the alpha to generate additional scenarios – lowering it means increasing the probabilities of large deviations and increasing it means reducing the probabilities. What would such a method reveal? It would certainly do what "sigma" and its siblings cannot do, which is to show how some portfolios are more robust than others to an entire spectrum of extreme risks. It can also show how some portfolios can benefit inordinately from wild uncertainty.

Despite the shortcomings of the bell curve, reliance on it is accelerating, and widening the gap between reality and standard tools of measurement. The consensus seems to be that any number is better than no number – even if it is wrong. Finance academia is too entrenched in the paradigm to stop calling it "an acceptable approximation".

Any attempts to refine the tools of modern portfolio theory by relaxing the bell curve assumptions, or by "fudging" and adding the occasional "jumps" will not be sufficient. We live in a world primarily driven by random jumps and tools designed for random walks address the wrong problem. It would be like tinkering with models of gases in an attempt to characterise them as solids and call them "a good approximation".

While scalable laws do not yet yield precise recipes, they have become an alternative way to view the world, and a methodology where large deviation and stressful events dominate the analysis instead of the other way around. We do not know of a more robust manner for decision-making in an uncertain world.

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