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Fractals, Graphics, & Mathematics Education
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Foreword by Lynn A. Steen

In 1980 Ronald Reagan was elected president of the United States, IBM began an urgent program to develop a personal computer for which Microsoft agreed to provide the operating system, and Benoit Mandelbrot got his first real look at the archetypal fractal, the eponymous Mandelbrot set. One hundred years from now, which of these events is more likely to be remembered as having had the greatest influence on science and human affairs?

At this moment, two decades later, personal computers seem to be well in the lead. PCs are on every desk, all now connected by the world-linking Internet. The personal computer has truly transformed the way the world works. No invention since the printing press has created such a widespread impact and no human activity has ever changed society so quickly.

However, when we examine Internet patterns in detail, guess what we find just beneath the surface? The footprints of fractals. These wondrous geometric objects, discovered by Mandelbrot just over twenty years ago, turn out to be the key to understanding the frenetic behavior of signals linking the world's computers, as well as the means to efficient compression that makes it possible to transmit images over the Internet. Without fractals, engineers would never have been able to make the World Wide Web work as well as it does. And without the Web, PCs would be just one more labor-saving appliance.

Computers are important instrumentally; they provide tools that enable us to work more efficiently, to see patterns previously hidden, and to organize information in new and revealing ways. In contrast, fractals are important fundamentally; they provide elements of a totally new geometry that offer a profoundly different way to understand nature. In the long run, this new understanding of nature will count for far more than momentary advances in technology or politics.

The predominant Western view of the relation between mathematics and nature is a legacy of Plato's distinction between a world of ideals and a world of actualities. Mathematics, in this view, belongs to the ethereal world of ideals; nature, being earthly rather than heavenly, belongs to a world of actualities that is both imperfect and incomplete. Thus, reality is best understood by approximation in terms of ideal mathematical models.

Our most important inheritance from this tradition is Euclidean geometry, the axiomatic study of lines, circles, and triangles that form an ideal (and therefore approximate) basis for understanding geography, mechanics, astronomy, and everything real. From this perspective, nature is like noisy mathematics—rough and crumpled, slightly out of focus.

Fractals create an alternative to Euclidean geometry whose elements are not lines and circles but iterations and self-similarities, whose surfaces are not smooth but jagged, whose features are not perfect but broken. Derived from apparent pathologies that puzzled or affronted traditional mathematicians, fractals reveal an entirely new geometry that enables us to understand formerly inexplicable real-world phenomena.

Fractals provide insight into the distribution of galaxies, the spread of bacterial colonies, the grammar of DNA, the shape of coastlines, changes in climate, development of hurricanes, growth of crystals, percolation of ground pollutants, turbulence of fluids, and the path of lightning. They have been employed to create powerful antennas, to develop fiber optics, to monitor financial data, to compress images, and to produce artificial landscapes. Their influence has been felt in art, architecture, drama (*Arcadia*), film (*Jurassic Park*), music, and poetry. Fractals have even penetrated the inner sanctum of elite culture—New Yorker cartoons.

For many, this extraordinary utility would be a sufficient warrant for fractals to be awarded a prominent role in mathematics education. But reasons other than utility can also be advanced, and it is these reasons, not utility, that form the major thrust of this volume.

Simply put, fractals enable everyone to enjoy mathematics. Nothing else can make such a striking—and important—claim.

Arguments about strategies for improving mathematics education roil state and local school politics. Some urge strict exam-enforced standards; others advocate more inviting contexts for learning. Some promote traditional curricula; others support enhanced or integrated programs. Rarely do the protagonists in these "math wars" stop to ask whether different mathematics might yield increased learning.

But that is precisely the argument advanced by the authors represented in this volume. They focus on teaching fractals, not primarily because fractals are important but because learning about fractals is, as one student put it, "indescribably exciting . . . and uniquely intriguing." It is easy to see why:

- The first steps are so much fun. Exploring fractals creates unprecedented enthusiasm for discovery learning among both teachers and students.

- Fractals are beautiful. Stunning visuals appeal to the mind's eye and create contagious demand for continued exploration.

- Anyone can play. Exploration of fractal geometry appeals to students of every age, from primary school through college and beyond.

- Fractals promote curiosity. Simple rules, easily modified, create nearly uncontrollable temptations to explore different options to see what surprising patterns will emerge.

- Simple ideas lead to unexpected complexity. Fractals are more life-like than objects studied in other parts of mathematics; thus they appeal to many students who find traditional mathematics cold and austere.

- Many easy problems remain unsolved. Fractals are rich in open conjectures that lead to deep mathematics. Moreover, the distance from elementary steps to unsolved problems is very short.

- Careful inspection yields immediate rewards. Insight and conjectures arise readily when our well-developed visual intuition is applied to fractal images. In studying fractals, children can see and conjecture as well as adults.

- Computers enhance learning. The visual impact of computer graphics makes fractal images unforgettable, while the unforgiving logical demands of computer programs yield important lessons in the value of rigorous thinking.

The history of mathematics education is long and convoluted, reflecting both the changing nature of mathematics and the evolving demands of society. Although in the eighteenth and nineteenth centuries mathematics was both experimental and theoretical, during much of the twentieth century the theoretical aspect has dominated. Much of mathematics education followed this trend towards theory and abstraction, leading to alarming reports of rising mathematical illiteracy not only in the United States but in many other countries as well.

Fractals represent a rebirth of experimental mathematics, enabled by computers and enhanced by powerful evidence of utility. In the ebb and flow of mathematical fashion, the struggle between theoretical and experimental is once again more nearly in balance. What remains is the challenge of restoring this balance to mathematics education. It is to that important task that this book is devoted.

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