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Fractal aggregates, and the current lines of their electrostatic potentials

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We have constructed diffusion limited aggregates off-lattice, that is, using Brownian motion instead of random walk, and the equipotential and the current lines have also been computed in the continuous plane. This construction (describes as being off off-lattice) is illustrated for the composite of a circular and a cylindrical DLA cluster. The striking new finding we report is that the current lines form trees with nearly straight branches.

For Michael E. Fisher on his sixtieth birthday. Flowers
do not last, and our sober culture unfortunately
restricts the conditions under which they can be offered.
But our culture has also given us a sober tool, the computer,
and in due time, the computer has brought
the eye and color back into the hard sciences.
Countless examples have shown that a physicist is
quite able to develop effective theories of objects that
he cannot see, or whose very existence is only an attractive but
unproven conjecture. In other words,
physics *can* develop without the eye. *But it does not have to.*
We had wanted, before we study diffusion limited aggregation
further, to be sure of what DLA clusters really look like.
The results have surprised and enchanted us,
and we offer one to you in lieu of flowers. *B.B.M.*

Our figure combines, in a somewhat fanciful manner, a circular and a cylindrical cluster of diffusion limited aggregation [1]. They were grown independently by continuous Brownian motion, and later were brought together and put at potentials 1 and 0, respectively. The Laplacian electrostatic potential was then computed and the current lines – defined as being orthogonal to the equipotentials – were drawn.

In practice, of course, "continuous" or off-lattice [2] means that the step length σ

of the Brownian movement of the aggregating atoms is much smaller than the diameter Δ of the atoms; we chose $\sigma = \Delta/100$. For consistency, the Laplacian potential is also estimated using a lattice with $\delta = \Delta/8$. The circular and cylindrical clusters contain respectively 659 and 1093 atoms.

The most important features of the potential around either of our clusters are not affected by the presence of the other cluster.

Observe that the particles are drawn as circles of diameter a bit larger than Δ, hence resulting in a slight overlap that creates the attractive and useful impression of "coral", or of branching polymer molecules. The coral's black background extends to a distance Δ from the particle centers. If the particles themselves were also made black, one would obtain the "Minkowski Δ sausage" (Δ-neighborhood) of the collection of particles centers. In our simulations, a new particle becomes attached to the cluster when its center comes within any distance smaller than σ of the boundary of this sausage.

What our figure tells us is best understood in light of earlier observations by Mandelbrot and Evertsz [3]. As is well known, DLA clusters include deep "fjords" characterized by fractal boundaries and apparent "necks". Yet, ref. [3] reports that the geometric roughness of the fractal boundary of the fjords and the necks has little direct effect on the shapes of the equipotentials. Thus, a typical fjord's potential exhibits a remarkably smooth "mainstream tree". Typically, the distance between the equipotentials does not oscillate, but decreases stepwise down this tree's branches, with each branch having near parallel walls. (The regions of fastest growth do not exhibit such completed mainstreams, but fan-shaped bays.)

The figure takes the next step, and illustrates the current lines that run from every sublattice point near each DLA cluster's boundary, and cross the equipotentials at right angles. These lines are obtained by releasing a massless charged particle from each boundary site of both clusters and following its trajectory until it hits the boundary of the oppositely charged cluster. The intensity of the lines is a function of the density of these trajectories.

We see that these current lines form another set of striking trees, and our main observation is that those trees' branches are very smooth and nearly straight. This reinforces very strongly the observations in ref. [3].

The observed shapes of the mainstream trees and of current trees are specific to DLA, and not "universal" (which would make them insignificant). For example, the potentials around other complex shapes (Julia sets or Mandelbrot sets) can take extremely different shapes [4]. The special potential or current trees of DLA must be due to the fact that the cluster itself grows in response to the potential.

By examining our figure, the mystery of DLA has been refined, but not yet solved.

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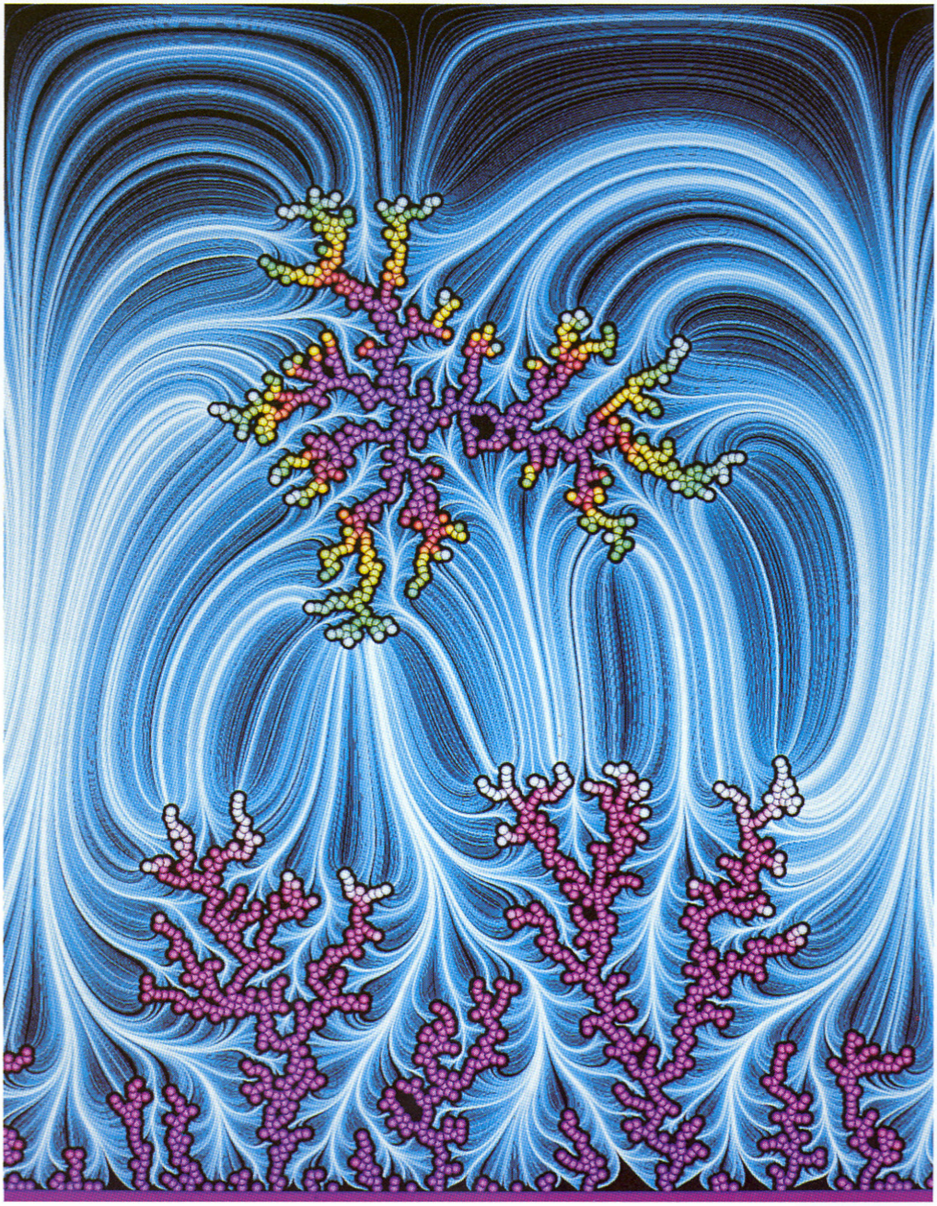

References

[1] T.A. Witten and L.M. Sander, Phys. Rev. Lett. 47 (1981) 1400.
[2] P. Meakin, in: Phase Transitions and Critical Phenomena, vol. 12, C. Domb and J. Lebowitz, eds. (Academic Press, New York, 1988) p. 335.
[3] B.B. Mandelbrot and C.J.G. Evertsz, Nature 348 (1990) 143.
[4] H.O. Peitgen and P.H. Richter, The Beauty of Fractals (Springer, Berlin, 1986).