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Fractal character of fracture surfaces of metals

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When a piece of metal is fractured either by tensile or impact loading (pulling or hitting), the fracture surface that is formed is rough and irregular. Its shape is affected by the metal's microstructure (such as grains, inclusions and precipitates, whose characteristic length is large relative to the atomic scale), as well as by 'macrostructural' influences (such as the size, the shape of the specimen, and the notch from which the fracture begins). However, repeated observation at various magnifications also reveals a variety of additional structures that fall between the 'micro' and the 'macro' and have not yet been described satisfactorily in a systematic manner. The experiments reported here reveal the existence of broad and clearly distinct zone of intermediate scales in which the structure is modelled very well by a fractal surface. A new method, slit island analysis, is introduced to estimate the basic quantity called the fractal dimension, *D*. The estimate is shown to agree with the value obtained by fracture profile analysis, a spectral method. Finally, *D* is shown to be a measure of toughness in metals.

Fractals are the concern of a new geometry[1], whose primary object is to describe the great variety of natural structures that are irregular, rough or fragmented, having irregularities of various sizes that bear a special 'scaling' relationship to one another. In very loose terms, they seem to fall into a regular hierarchy in which each level is an up-sized or down-sized version of the levels below or above it. Fractal geometry characterizes the scaling structure of a surface by a number D, called the fractal dimension, that can range from 2, when the surface is smooth, up to 3. The fractal dimensional increment of a surface is $D-2$. As it increases from 0 to 1, the irregularities become increasingly predominant and the notion of overall shape of the surface becomes progressively less meaningful. Similarly, the fractal dimension of a curve is at least 1, the fractal dimensional increment being $D-1$.

The term 'fractal' was chosen in explicit cognizance of the fact that the irregularities found in fractal sets are often strikingly reminiscent of fracture surfaces in metals (though not, for example in glass). However, metal fractures are only extremely crinkly (down to the limits of their microstructural size range), while fractals are infinitely crinkly. Hence, it is not possible to say that metal fracture is strictly a fractal. Nevertheless, metals resemble a fractal so closely that it makes good sense to use a metal for modelling a fractal. Our experiments in metal fracture show that D is very well defined for all the specimens examined, and takes on the same value for different specimens of the same metal having similar thermomechanical treatments.

Recent studies[2,3] conclude that a metal fracture's spectrum is indicative of a fractal structure. A recent review[4] also indicates that fractals might model metal fracture surfaces, and reports exploratory studies. The more refined tools that we use to analyse metal fracture surfaces make a stronger case and add to the existing tools of fractographic analysis[4,5].

Our first series of tests is exemplified by Fig. 1. A fractured steel specimen was plated with electroless nickel and mounted in an epoxy mount by vacuum impregnation in order to ensure edge retention. The specimen was then polished parallel to the plane of fracture. 'Islands' of steel surrounded by nickel appeared which, on subsequent polishing, grew and merged. We propose to call these structures slit islands. The islands contain 'lakes within islands' and 'islands within lakes'; we include the former and neglect the latter.

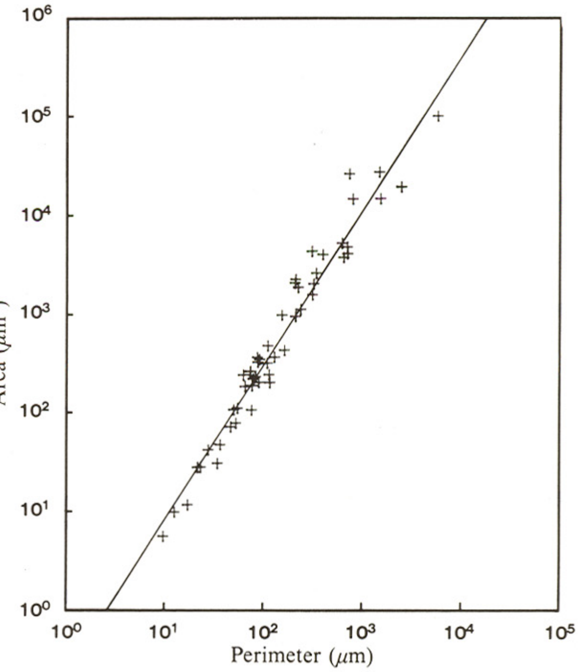

Fig. 1 Fractal area–perimeter relationship for slit islands. 300-Grade Maraging steel. Ruler = 1.5625 μm; fractal dimensional increment = 1.28.

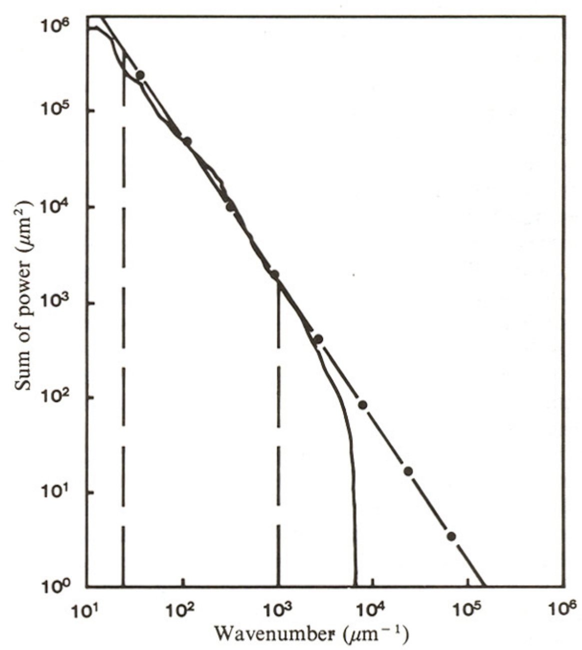

Fig. 2 Cumulative spectrum for vertical cuts. 300-Grade Maraging steel. Slope = 1.4878; fractal dimensional increment = 1.26. Average of five profiles.

The slit island 'coastlines', being curves, are much easier to investigate than the surfaces themselves. Yet we have found that the perimeter–area relationship of fractal dimensional analysis (ref. 1, Chap. 12), if applied to these coastlines, reveals many essential facts about the surface. We propose that the resulting method of analysis, which is entirely new and very powerful, be called slit island analysis. When islands are derived from an initial fractal surface of dimension D by sectioning with a plane, their coastlines are of fractal dimension $D' = D-1$. Thus, the fractal dimensional increments $D-2$ and $D'-1$ are equal. The theory of fractals suggests that the areas and

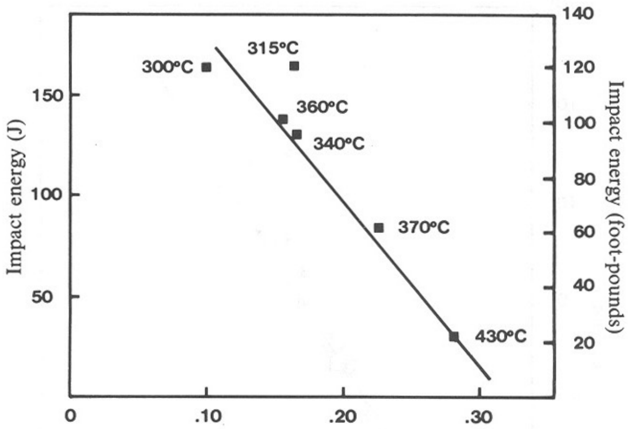

Fig. 3 Impact energy versus fractal dimensional increment. 300-Grade Maraging steel, room temperature, Charpy impact.

perimeters of these islands should be measured in the same way, and that one should trace the graph of log(perimeter) versus log(area). In a fractal, this graph is rectilinear of slope D'. Figure 1 shows that such is indeed the case for fracture surfaces.

Our second series of tests of fractal structure which complements the first, uses fracture profile analysis based on Fourier analysis[3] (Fig. 2). Here, the plated and processed fracture was sectioned perpendicular to the fracture surface to expose it in profile. As usual, the profile's sample spectra exhibit wide oscillations. Some are statistical artefacts, but others reflect fundamental lengths of the microstructure and their accompanying higher order harmonics. Statistical fluctuations must be averaged out and the fluctuations due to the microstructure should be prevented from overly affecting the analysis. To achieve both goals, we averaged five spectra taken from serial sections, and integrated them from high frequency to low. Now, the fractal hypothesis requires the integrated spectrum to take the form $k^{-B'}$, with $B'=B-1=6-2D$. The fractal character of the surface was tested by plotting the data on doubly logarithmic coordinates to see if the curve has a large straight central portion of absolute slope B'. The plots (Fig. 2) are indeed straight but over narrower ranges than the area–perimeter plots. Thus, the fracture surface is again inferred to be fractal, and its fractal dimension is estimated by $D=3-B'/2$.

When a method of analysis yields a nearly straight graph, it is tempting to measure the 'local' slope of this curve. The goal would be to identify the harmonics of the microstructure and other features that (contrary to the overall fractal behaviour) are defined by well-defined length scales. However, as the local slope is severely affected by statistical fluctuations, attempts to interpret its variations are useless. Similarly, the 'spectrum of fractal dimension' described in ref. 5 mostly reflects the underlying noise.

Careful analysis of some of our data indicates that the central region of the log–log plots splits into two distinct subregions characterized by different values for D. Such a juxtaposition of different fractal zones is very familiar in other applications of fractals. The scales of the clear-cut cross-over points between zones tend to be significant with respect to structure and deserve further investigation. For other metals, the diagrams are even more complex.

To relate D to known metallurgical quantities, we took a series of identical 300 grade Maraging steel Charpy impact specimens, and heat-treated them at different temperatures. Figure 3 shows that the value of D decreases smoothly with an increase of the impact energy as measured by a standard Charpy impact test. This relationship must reflect the changes in the microstructure that occur during ageing.

In our view, a fracture that is transgranular (that is, runs through the grains) involves an atypical form of the notion of percolation. For example, during fracture of a ductile material, any voids forming around inclusions increase in size and coalesce into void sheets; these ultimately form the fracture surface. If the growth of a void were independent of its neighbours or its position in the specimen, we would be faced with a process physicists call percolation. Its applicability here is associated with load sharing among the tensile ligaments that remain; void sheet coalescence is merely percolation of a crack in the microstructure leading to final fracture. Percolation would imply that the fracture's fractal dimension takes some universal value independent of the material, and dependent only on the properties of space. But percolation is a very crude model here. As soon as the initial void growth has coalesced locally into small sheets, the strains on the supporting ligaments increase, and surrounding voids grow at a rate that varies with their position in the specimen. Certainly, spatial variability associated with the microstructure is dependent on the microstructure and the resulting process differs from the usual percolation. Were this view to be confirmed by further experimentation, the study of fracture could benefit from techniques used in the study of phase transitions in statistical physics.

By showing linear relationships among all available data while covering such a broad range of sizes, Figs 1 and 2 are almost unique in metallurgy. Their linearity demonstrates that the fracture surfaces present in several grades of steel produced by impact or uniaxial tensile loading are fractal.

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