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SCALEBOUND OR SCALING SHAPES: A USEFUL DISTINCTION IN THE VISUAL ARTS AND IN THE NATURAL SCIENCES

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It is often said that 20th-century 'modern' buildings are sterile, not built to human scale and, in fact, unnatural. The more I ponder such statements and their variants, the more elusive I find their meaning and the more I feel that a discussion of the logic and of the aesthetics of the notion of *scale* needs to be resumed. Clearly, a building's absolute or even relative height and number of stories are incidental and other aspects of scale are more important: I propose that it might be interesting to introduce into aesthetics the distinction between *scalebound* objects and *scaling* objects, a broad distinction that is proving increasingly useful in several scientific contexts. One of my conclusions is that it is fruitful to call Mies van der Rohe's buildings scalebound—a term a physicist would use to describe a flawless crystal and the solar system—and to call the Paris Opera House a scaling building—the term scaling also being applicable to typical views of the Alps and to the visual characteristics of many other objects in nature, some of them visible (large or small) and others invisible to the naked eye.

Before I elaborate on this dichotomy, I must emphasize it has limits. I realize that numerous examples both in the sciences and in the visual arts combine scalebound and scaling features. In addition, one must keep in mind the pitfalls of analogy; mine are unlikely to be entirely new and are certain to be found eventually unsatisfactory through counterexample and contradiction.

In very rough outline (important features will be introduced gradually later), I propose the term *scalebound* to denote any object, whether in nature or one made by an engineer or an artist, for which characteristic elements of scale, such as length and width, are few in number and each with a clearly distinct size.

A spherical radome for sheltering a radar assembly is particularly scalebound, since its perceived scale is determined by the smallest possible number of measurements: a single intrinsic measure, its radius, and a single extrinsic one, the distance from which it is viewed; if a scalebound object is on the human scale, it will not, in general, be on the scale of a fly. Consider another good example: a Bauhaus style, glass cube-type building. If its windows are identical rectangles delimited by metal grids, its only characteristic lengths are the height and width of the whole and of the windows, where the smaller is a harmonic (whole divisor) of the bigger fundamental one. Its basic structure is so spare that as the result of slight changes, such as interrupting the façade by a few floors of different design or by changing the divisions between the windows from thin metal to thick concrete, the overall structure is enormously enriched. Some Post-Bauhaus architects are experts at such enrichment. In any event, any scalebound object does have a scale of its own, and the nearest it can come to being on a human scale is when it has the same size as a human standing next to it.

A scaling object, by contrast, includes as its defining characteristic the presence of very many different elements whose scales are of any imaginable size. There are so many different scales, and their harmonics are so interlaced and interact so confusingly that they are not really distinct from each other, but merge into a continuum. For practical purposes, a scaling object does *not* have a scale that characterizes it. Its scales vary also depending upon the viewing points of beholders. The same scaling object may be considered as being of a human's dimension or of a fly's dimension.

In several natural sciences, developing independently of each other, the distinction between scalebound and scaling objects has recently acquired much importance. For example, this distinction is basic to my analysis of shapes that I call *fractal* sets. This leads to fractal geometry, a new geometry arising next to Euclid and devoted to shapes with many scales of length [1]. In addition, I believe that fractals can also be useful in the study of diverse types of visual art, and I hope this Note will arouse discussions of this aspect of fractals. For each side of the dichotomy between scalebound and scaling, objects of art and of nature will be examined in turn.

I think it is evident that in Occidental traditional compositions of visual art, as exemplified by representational figurative paintings and television productions and traditional architecture of the beaux-arts kind, the extent of scaling is both considerable and deliberate. Two strict scales are of course present in the case of paintings: A maximal scale or outer cutoff is imposed by the size of the canvas and a minimal scale or inner cutoff is imposed by the width of brush strokes. But it seems typical that all scales between these limits should be present in depicted features scattered in various places on the pictorial plane. To test this last assertion qualitatively, it suffices to take a photographic negative of such a painting, cut it haphazardly into several different parts and print each part on a piece of paper of the same prescribed size. Some prints will contain almost nothing of note (this feature is an important facet of *non-trivial scaling*, and I shall return to it). But the prints that are not empty will tend to contain

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roughly the same amount of detail, as if the original included local latent details of many sizes. Small non-empty parts of a picture seem to echo its whole, but in a different detail and on a smaller scale. In architecture, the same comparison-of-cut-up-prints test is equally positive for a building regarded as aesthetically excellent and for one regarded as grossly poor. As to television, one reason I include it is that my subjective judgement of 'sameness' of the amount of detail in the succession of pictorial compositions on a screen is confirmed by quantitative tests unknowingly carried out by engineers. (For the benefit of the specialists among my readers, the indirect but convincing evidence that television pictorial compositions are scaling is described by stating that the signals used to transmit these pictures are *scaling noises*; they have a spectral density inversely proportional to frequency.)

In parallel to its occurrence in visual art, design architecture, television, etc., the property of scaling also shows up in many of the most readily perceived aspects of nature. Some of these aspects have attracted the attention of humans from their earliest days, and a few of them are mentioned in the Bible, for example the clustering in the distribution of stars. The story of Joseph, son of Jacob and Pharaoh's adviser is also (as I have shown elsewhere) best understood in terms of scaling. And many aspects of clouds and of weather involve scaling; witness the role of this concept in the study of fluid turbulence. But the most visible examples of geometric scaling characteristics are those inherent to mountains, lakes and seacoasts, each viewed from the appropriate distance (a big portion has to be viewed from an airplane, a smaller one from a tower, a still smaller one from ground level) (Fig. 1). They do differ in detail, but in overall structure they look very much the same. This observation suggests that it makes at least a little sense to consider that the role of scaling in representational figurative and nonfigurative pictures could well be influenced by the role of scaling in nature. After all, at least part of visual art does consist of depictions of either natural or human-made objects.

Incidentally, one is faced here with an instance where scientists' and artists' concepts of what is aesthetically satisfying are in sharp conflict. Variety of scales makes representational figurative and nonfigurative paintings both attractive and demanding in terms of particular skills of depiction, but, by the same token, variety of scales makes the more readily perceived aspects of objects in nature horrendously difficult to study scientifically. It is at least in part responsible for the slowness of scientists in attempting such studies and for the fact that the results obtained have been much less complete.

For contrast, I shall return to the consideration of scalebound objects. As I said, particularly obvious examples are found in the buildings designed by Mies van der Rohe. The cut-up-prints test, mentioned above, yields readily distinguishable individual prints. The same is even more evident if the test is applied to typical products of advanced technology. And there is no need to stress that when engineers' ultilitarian purposes lead them to change complex scaling objects of nature, for example the shapes of coastlines and the courses of rivers, they often simplify and straighten them out, in other words bind them to scales that suit humans.

I do not imply that there are no scalebound objects in nature. Some crystals are good examples, but somehow they do seem less 'natural' than the complex shape of a coastline. Scientists have discovered other examples of

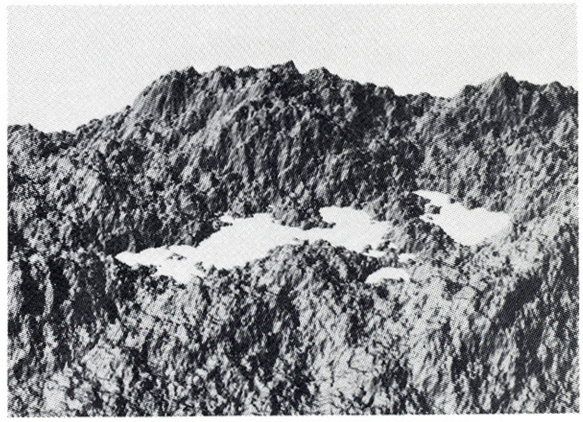

Fig. 1. Richard F. Voss. Computer generated fractal landscape made with an algorithm by B. B. Mandelbrot. (A scientific illustration that exhibits the coexistence of all scales.)

simple scalebound objects in nature and have organized them as parts of an increasingly complex whole. Furthermore, these aspects of objects in nature have had numerous practical applications. However, the impressive edifice of physics should not distract one from the fact that physicists deal with an extremely small portion of nature. They acknowledge that one of the main reasons for the success of their science resides in the fact that they restrict themselves to comparatively simple scalebound objects.

I had thought for a long time that the relationship between the natural and the human-made is nearly reversed in the case of classical European music. Its structure had seemed to me to be scalebound (perhaps less so for a symphony than for a flute solo), while natural noise, a limit approached by certain contemporary music, would be a scaling structure. However, Richard Voss's recent studies show that classical music, after all, also has a scaling structure [2].

Now I shall return to the importance of near empty portions to the whole of a nontrivially scaling object. (A uniform field of dots is scaling too, in a way, but it is called trivial because one does not regard it as interesting.) When I said that a coastline viewed from different distances looks much the same, I omitted to emphasize that most of these views show little or nothing of interest.

Similarly, a typical Renaissance portrait depicts a seemingly obligatory juxtaposition of featureless areas (including draperies) and of finely detailed areas (including background landscapes that may often seem to lack any other purpose). The featureless portion of the whole view is variable in area, usually neither too small nor too large, often one- to two-thirds of the whole. Of course, the largest feature is highly dependent upon the distance between the portrait and its viewer. As one gets closer to an interesting feature, details of constantly smaller absolute size become the largest visible, and at the same time, the details that used to be the most significant become even larger and, eventually, move out of view. If, on the contrary, one gets closer to a featureless area of the portrait, one soon ceases to see anything of note. Similarly, it is indeed an important characteristic of a typical scaling pattern in nature that a portion selected without a systematic bias tends to include substantial featureless portions. Incidentally, while the maximum size of a painting is that of the whole canvas, many painters succeed in giving the impression that it contains depicted objects of larger sizes. Similarly, a brush stroke determines the minimum size, but many portrait jewels seem to include detail that is known to be physically impossible to depict at the scale depicted. Thus, some portraitists work to give the impression of having extended the range of scaling beyond the limits imposed by the size of a canvas and of a brush stroke. Finally, pictures on a cabinet television screen also exhibit these characteristics and, in addition, move rapidly through the grossly nonscaling scenes (such as crowd scenes) in which much that is significant is at the same time smaller than the frame and larger than the least perceptible detail.

References and Notes

1. B. B. Mandelbrot, *Fractals: Form, Chance, and Dimension* (San Francisco: W. H. Freeman, 1977). A review of the book appears in Books, *Leonardo* **12,** 248 (1979).
2. M. Gardner, White and Brown Music, Fractal Curves and one-over-f Fluctuations, *Scientific American,* p. 17 (April 1978).
3. This note was submitted to *Leonardo* in draft form in 1971, but its actual completion was delayed because I was working on different aspects of the subject, as presented in my book listed in Ref. 1, above. I regard this Note as a mere sketch, and I hope to expand the analysis when an opportunity arises.