

ON INTERMITTENT FREE TURBULENCE

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The distribution of the rate of dissipation of free turbulence, in the ocean and the atmosphere, seldom satisfies the homogeneity assumption of the classic Kolmogoroff-Obukhov theory. Analogous "intermittency" (with "clustering" of the "active regions") is also observed for the energy of various "1:f noises," for the spatial mass distribution of rare resources such as metals, and elsewhere.

By putting together the discussions of the various intermittent phenomena, one finds two broad approaches. The first was followed by de Wijs (a geophysicist), and later by Obukhov, Kolmogoroff and Yaglom (in *Boundary Layers and Turbulence*, Proceedings of the 1966 Kyoto Symposium, *Physics of Fluids*, Vol. 10, 1967). The second approach was followed by Mandelbrot (who started in the context of 1:f noises) and by Novikov and Stewart (see Mandelbrot's Kyoto paper, loc. cit.)

The purpose of the present work has been to construct further variants of both broad approaches to intermittency, to unify all the approaches, and to develop them.

Of particular interest is that all approaches suggest—in agreement with experience—that any interval containing turbulent energy, when examined more closely, will be found to include inserts effectively devoid of turbulence. Mathematically, this may be expressed by saying that turbulence is not carried by intervals, but by "thin" sets with many gaps. Such sets can be characterized as having a "dimension" d less than unity (for a discussion of fractional dimension, see Mandelbrot's report in *Science*, May 5, 1967). At one extreme, when there is no insert, $d = 1$; at the other extreme, where turbulence is concentrated within a single "puff," $d = 0$. Another property of intermittency is the exponent Ω that enters in the expression $k^{-\Omega}$, which is proportional to the spectral density of the rate of dissipation ε . Different approaches to turbulence yield different relations between d and Ω . This suggests, either that further assumptions are needed to identify the properties of "the" actual turbulence, or that different models of intermittency are to be used in different contexts.

The random variable $\varepsilon(r)$, the average dissipation in a sphere of radius r , is never found to be lognormally distributed (which denies a hasty claim of de Wijs, Kolmogoroff, Obukhov and Yaglom). Moments of $\varepsilon(r)$ are obtained. In some cases $\varepsilon(r)$ is found to have infinite population variance. This implies—in agreement with experience—that observed values of $\varepsilon(r)$ can be very widely scattered.

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