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Sporadic Turbulence

BENOIT MANDELBROT
Thomas J. Watson Research Center, International Business Machines Corporation
Yorktown Heights, New York
and
Massachusetts Institute of Technology, Cambridge, Massachusetts

Turbulence in the atmosphere and the ocean, and often in the laboratory, is "spotty" or "intermittent"; it does not satisfy the homogeneity assumptions of the 1941 Kolmogoroff–Obukhov theory. Moreoever, Landau and Lifshitz soon pointed out that ϵ_r, the mean rate of energy dissipation in a volume of radius r, must be a random variable whose law depends upon the ratio of r to the external scale L_e. In 1961 Obukhov gave a theory of intermittency, later developed by Kolmogoroff and Yaglom, that leads to the lognormal law for ϵ_r. Sporadic turbulence is an alternative to Obukhov's theory and is akin to the theory of intermittency due to Novikov and Stewart.[1]

Sporadic turbulence is extremely intermittent. Like some approaches to homogeneous turbulence, its theory involves no physical "cascade" argument, and is based exclusively upon axioms of self-similarity, dimensional correctness, and local character. The difference is that the axioms are geared to turbulent-laminar mixtures, by always being stated for conditional (rather than for absolute) probability distributions. "Ensemble expectations" are not unique, but depend upon a "conditioning event," so that the ergodic problem is very complex.[2] For example, one may consider Pr $[u(x+r) - u(x) \neq 0$, given that $u(0) - u(L) \neq 0$ and $0 < x < x + r < L]$. By self-similarity, this must take the form $(r/L)^\theta$, where $0 \leq \theta \leq 1$. θ is a measure of "degree of spottiness": $\theta = 1$ in the homogeneous case, and $\theta = 0$ when all turbulent energy is concentrated in a single "puff." Kolmogoroff's law, $\langle [u(x+r) - u(x)]^2 \rangle = Cr^{\frac{2}{3}}$, only applies to the expectation taken under the condition "$u(y)$ is not constant for $x < y < x + r$." The alternative "laminar" solution, $\langle [u(x+r) - u(x)]^2 \rangle = 0$, also satisfies Kolmogoroff's axioms, applies if $u(y) \equiv$ constant and plays a central role in sporadic turbulence. For example, whenever $0 < x < x + r < L$, one has $\langle [u(x+r) - u(x)]^2 \mid u(y)$ non-constant for $0 < y < L \rangle = C\epsilon_L^{\frac{2}{3}} L^{(\theta-1)/3} r^{1-\theta/3}$.

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[1] E. A. Novikov and R. W. Stewart, Izv. Akad. Nauk SSSR, Ser. Geofis. 408 (1964).

[2] The underlying theory of sporadic random functions is described in: B. Mandelbrot, in *Proceedings of the Fifth Berkeley Symposium of Mathematics and Statistical Probability* (to be published); also IEEE Trans. Inf. Theory (to be published).