quick study

The universe in a cup of coffee

John S. Wettlaufer

Your morning java or tea is a rotating, cooling laboratory that reflects the physics of such large-scale phenomena as stellar dynamics and energy transport in Earth's atmosphere and oceans.

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As people throughout the world awake, millions of them every minute perform the apparently banal act of pouring cold milk into hot coffee or tea. Those too groggy to reach for a spoon might notice upwelling billows of milk separated by small, sinking, linear dark features such as shown in panel a of the figure. The phenomenon is such a common part of our lives that even scientists—trained to be observant—may overlook its importance and generality. The pattern bears resemblance to satellite images of ocean color, and the physics behind it is responsible for the granulated structure of the Sun and other cosmic objects less amenable to scrutiny.

Archimedes pondered the powerful agent of motion known as buoyancy more than two millennia ago. Children do, too, when they imagine the origins of cloud animals on a summer's day. The scientific study of thermal and compositional buoyancy originated in 1798 with a report by Count Rumford intended to disabuse believers of the caloric theory. Nowadays, buoyancy is at the heart of some of the most challenging problems in nonlinear physics—problems that are increasingly compelling. Answers to fundamental questions being investigated today will have implications for understanding Earth's heat budget, the transport of atmospheric and oceanographic energy, and, as a corollary, the climate and fate of stars and the origins of planets. Few avenues of study combine such basic challenges with such a broad swath of implications. Nonetheless, the richness of fluid flow is rarely found in undergraduate physics courses.

Wake up and smell the physics

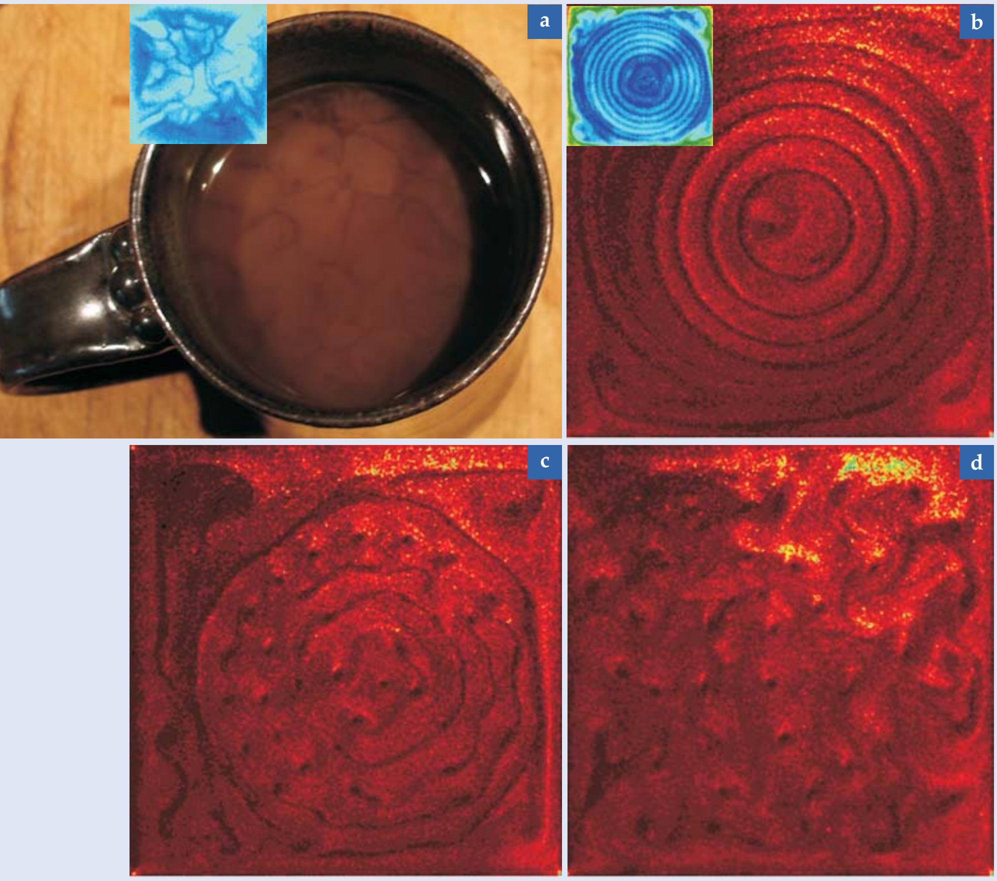
The modern theory of hydrodynamic stability arose from experiments by Henri Bénard, who heated, from below, a thin horizontal layer of spermaceti, a viscous, fluid wax. For small vertical temperature gradients, Bénard observed nothing remarkable; the fluid conducted heat up through its surface but exhibited no wholesale motion as it did so. However, when the gradient reached a critical value, a hexagonal pattern abruptly appeared as organized convective motions emerged from what had been a homogenous fluid. The threshold temperature gradient was described by Lord Rayleigh as reflecting the balance between thermal buoyancy and viscous stresses, embodied in a dimensionless parameter now called the Rayleigh number. When the momentary thermal buoyancy of a blob of fluid—provided by the hot lower boundary—overcomes the viscous stresses of the surrounding fluid, wholesale organized motion ensues. The strikingly structured fluid, with its up-and-down flow assuming specific geometries, is an iconic manifestation of how a dissipative system can demonstrate symmetry breaking (the up-and-down flow distinguishes horizontal positions even though the lower boundary is at a uniform temperature), self-organization, and beauty. (See the article by Leo Kadanoff in PHYSICS TODAY, August 2001, page 34.)

Astrophysicists and geophysicists can hardly make traction on many of the problems they face unless they come to grips with convection—and their quests are substantially complicated by their systems' rotations. Despite the 1835 publication of Gaspard-Gustave Coriolis's *Mémoire sur les équations du mouvement relatif des systèmes de corps* (*On the Equations of Relative Motion of a System of Bodies*), debate on the underlying mechanism behind the deflection of the Foucault pendulum raged in the 1905 volume of *Annalen der Physik*, the same volume in which Albert Einstein introduced the world to special relativity. Maybe the lack of comprehension is not so surprising: Undergraduates still more easily grasp Einstein's theory than the Coriolis effect, which is essential for understanding why, viewed from above, atmospheric circulation around a low pressure system over a US city is counterclockwise but circulation over an Australian city is clockwise.

Practitioners of rotating-fluid mechanics generally credit mathematical physicist Vagn Walfrid Ekman for putting things in the modern framework, in another key paper from 1905. Several years earlier, during his famous *Fram* expedition, explorer Fridtjof Nansen had observed that ice floes moved to the right of the wind that imparted momentum to them. Nansen then suggested to Ekman that he investigate the matter theoretically. That the deflection was due to the ocean's rotating with Earth was obvious, but Ekman described the corrections that must be implemented in a noninertial reference frame. Since so much in the extraterrestrial realm is spinning, scientists taken by cosmological objects eventually embraced Ekman's formulation and sought evidence for large-scale vortex structures in the accretion disks around stars. Vortices don't require convection and when convection is part of a vortex-producing system, additional and unexpected patterns ensue.

Cream, sugar, and spinning

The Arctic Ocean freezes, cooling and driving salt into the surface layers. Earth's inner core solidifies, leaving a buoyant, iron-depleted metal. Rapidly rising air from heated land surfaces creates thunderstorms. Planetary accretion disks receive radiation from their central stars. In all these systems, rotation has a hand in the fate of rising or sinking fluid. What about your steaming cup of coffee: What happens when you spin that?



A cuppa vortex structures. (a) Everyone knows that if you wait for a while coffee will get cold. The primary agent doing the cooling is evaporatively driven convection. Pour cold milk into hot coffee and wait. The cold milk mixes very little as it sinks to the bottom of the cup, but eventually cold plumes created by evaporation at the surface sink down and displace the milk. In time, a pattern forms of upwelling (lighter) and downwelling (darker) fluid. The inset is an IR image of the surface of water evaporating at room temperature; the dark regions are a few tenths of a kelvin colder than the light regions. **(b)–(d)** Several views of a volume of water 11.4 cm deep with a cross section of 22.9×22.9 cm. Panel b shows the liquid about 7.5 minutes after the fluid is set in motion at a few tenths of a radian per second. The principal image indicates particle density (light is denser) at a depth of 0.6 cm below the surface. The inset is a thermal image of the surface. Panel c shows the breakup of the rings, 11 minutes after the initiation of rotation, due to a shearing instability. As panel d shows, at 14 minutes the breakup leads to a grid of vortices. A video showing the process is online at http://prl.aps.org/supplemental/PRL/v105/i4/e044504. (Adapted from J.-Q. Zhong, M. D. Patterson, J. S. Wettlaufer, *Phys. Rev. Lett.* **105**, 044504, 2010.)

Place the cup in the center of a spinning record player—some readers may even remember listening to music on one of those. The friction from the wall of the cup transmits stresses into the fluid interior. If the coffee is maintained at a fixed temperature for about a minute, every parcel of fluid will move at the same angular velocity; the coffee is said to be spun up.

On the time scales of contemporary atmospheric and oceanographic phenomena, Earth's rotation is indeed a constant, whereas the time variation of the rotation could be important for phenomena in planetary interiors, the evolution of an accretion disk, or tidal perturbations of a distant moon. Thus convective vortices are contemplated relative to a rotating background flow. Perturbations in the rotation rate revive the role of boundary friction and substantially influence the interior circulation. Moreover, evaporation and freezing represent additional perturbations, which alter how the fluid behaves as stresses attempt to enforce uniform rotation. Returning to the coffee mug as laboratory, the model system shown in panel b of the figure reveals how the added complexity of rotation momentarily organizes the pattern seen in panel a into concentric rings of cold and warm fluid.

Fundamental competitions play out when you rotate your evaporating coffee. As we have seen, evaporative cooling drives narrow regions of downward convection; significant viscous and Coriolis effects balance each other in those downwelling regions. Rotation then dramatically organizes the sinking cold sheets and rising warm billows into concentric rings that first form at the center of the cup. By about 7.5 minutes after rotation has been initiated, the rings shown in panel b have grown to cover most of the horizontal plane. Their uniform azimuthal motion exists for about 3.5 minutes, at which time so-called Kelvin–Helmholtz billows associated with the shearing between the rings appear at their boundaries, grow, and roll up into vortices; see panel c. Three minutes later, as shown in panel d, those vortices lose their azimuthal symmetry and assemble into a regular vortex grid whose centers contain sinking fluid.

Panel d shows one type of coherent structure that forms in rotating fluids and other mathematically analogous systems if the persistence time of the structure—vortices here—is much longer than the rotational period. Other well-known examples are Jupiter's Great Red Spot, which is an enduring feature of the chaotic Jovian atmosphere, and the meandering jet streams on Earth.

Moreover, persistent vortices in superconductors and superfluids organize themselves. Indeed, it appears that vortices in superconductors are as mobile as their counterparts in inviscid fluids. And although scientists have long studied rotating convective superfluids, the classical systems considered in this Quick Study suggest that we may yet find surprising analogies in superconductors. Will we one day see superconducting jet streams?

If you are reading this article with a cup of coffee, put it down and take a closer look at what is going on in your cup.

The online version of this Quick Study includes a short supplemental essay on cosmic vortices.

Additional resources

▶ S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*, Dover, New York (1981).

▶ D. Bercovici, "Mantle Dynamics Past, Present and Future: An Introduction and Overview," in *Treatise on Geophysics*, vol. 7, *Mantle Dynamics*, D. Bercovici, ed., Elsevier, New York (2007), p. 1.

▶ E. A. Spiegel, "Cosmic Vortices in Hot Stars and Cool Disks," *Theor. Comput. Fluid Dyn.* **24**, 77 (2010). ■

Cartesian vortex theory, cosmic vortices, and the route to cosmogony

John S. Wettlaufer

Supplemental material for the Quick Study, "The Universe in a Cup of Coffee."

The foundation of René Descartes's mechanics was torn asunder during early-18th-century debates in the Paris Academy of Sciences. The Cartesians constructed intuitive mechanical descriptions of the universe based on the three elements of primitive matter as devised by Descartes. One was the "subtle minute matter" constituting the Sun and the stars. Capable of moving at great speed, that form of matter permeated all space not occupied by the two other, denser elements. The second and principal form of celestial matter, spherical ether particles, moved in large vortical structures and transmitted force as an instantaneous impulse. The coarsest and most lethargic element composed planetary bodies, including Earth.

In the Cartesian view, the mechanics of the solar system was controlled by the action of the ether particles, through their impulsive coupling to planetary bodies. In Descartes's cosmogony, the guiding hands of vortices set in motion by the creator continually returned bodies to their proper orbits. The idea persisted because the Cartesians convincingly argued that the action of the ether particles mimicked that of the central force of gravity described by Isaac Newton and Gottfried Leibniz. Eventually, the Cartesian vortices vanished in their own ether, as they failed to describe the world as powerfully as did Newtonian mechanics.

A few centuries after the debates in the Paris Academy, Carl Friedrich von Weizsäcker developed a cosmogony that embraced vortices—but his mid-20th-century vortices emerged during the evolution of an accretion disk around a central star. Weizsäcker laid down the foundations of what we now understand is a multistage process that plays out over millions of years: Gas collapses, the accretion disk comes into being, mass accretes, primary planetesimals form, and a UV flux from the central star removes the lightest elements from the outer regions of the disk. The vortices arise in Weizsäcker's system because, on the one hand, viscous stresses act to enforce uniform rotation, but on the other, the Keplerian velocity—the speed determined by equating centripetal and gravitational force—falls with distance from the central star. Thus, the viscous forces decelerate the inner regions of a disk and accelerate the outer regions; the result is a state of quasi-stable vortex structures as shown in panel a of the figure. Weizsäcker surmised that such vortices act to focus primordial matter for gravitational collapse into protoplanets.

Scientists are working to formulate the precise mechanisms for agglomeration on scales too small for gravitational collapse and to explain how those mechanisms lead to gravitationally controlled growth. And in the decades since Weizsäcker's work, we have learned that accretion disks are rather less peaceful than in his model. Turbulence abounds and vortices arise from shear, magnetorotational, and so-called streaming instabilities and other sources; panel b of the figure shows a simulation. Nonetheless, in the current understanding, vortices play the same qualitative role as in Weizsäcker's vision: They are the future homes of consolidated matter. Vortices may be intrinsically transient, but they seem to persist in theories of cosmogony.

I am grateful to Edward A. Spiegel for reading an early version of this essay and offering helpful comments.

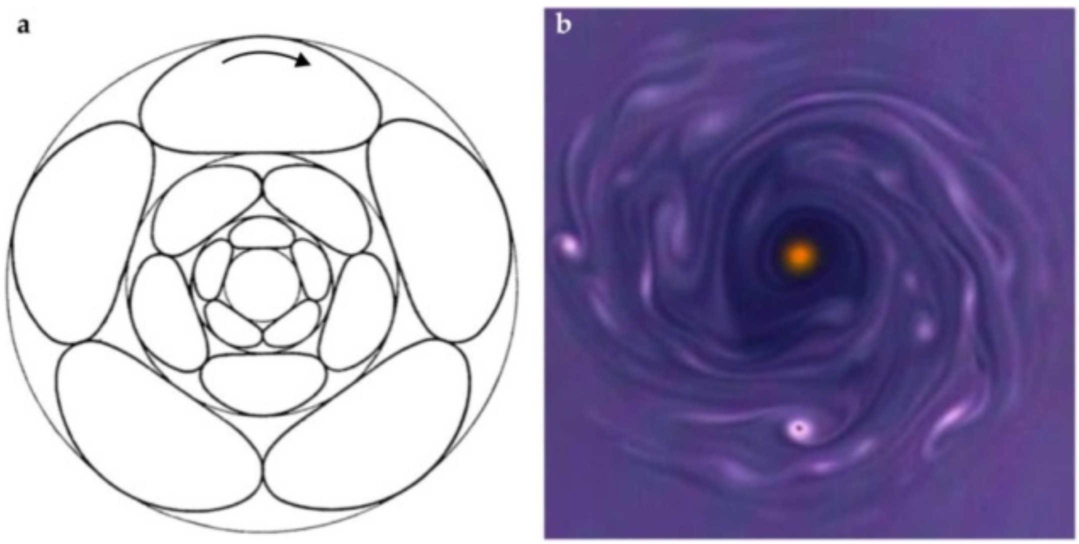


Figure caption: **Vortices in a protoplanetary accretion disk. (a)** Carl Friedrich von Weizsäcker's vision of vortices that arise as viscous stresses try to establish uniform rotation. If the overall rotation of the disk is counterclockwise, the relative motion of the vortex at the top is clockwise, as indicated by the arrow. **(b)** This simulation, by Annalisa Bracco and colleagues, shows vortices in a disk with an artist's conception of a bright star (orange) at the center. The vortex formation in the modern simulation is much less orderly than in Weizsäcker's model. (Images adapted from E. A. Spiegel, "Cosmic Vortices in Hot Stars and Cool Disks," in Special Issue, "150 Years of Vortex Dynamics," *Theor. Comput. Fluid Dyn.* **24**, 2010, p. 77.)

Additional Supplemental Resources

‣ C. F. von Weizsäcker, "Über die Entstehung des Planetensystems," *Z. Astrophys.* **22**, 319 (1943).

‣ S. Chandrasekhar, "On a New Theory of Weizsäcker on the Origin of the Solar System," *Rev. Mod. Phys.* **18**, 94 (1946).

‣ E. J. Aiton, *The Vortex Theory of Planetary Motions*, American Elsevier, New York (1972).

‣ C. Iltis, "The Decline of Cartesianism in Mechanics: The Leibnizian–Cartesian Debates," *Isis* **64**, 356 (1973).

‣ P. J. Armitage, *Astrophysics of Planet Formation*, Cambridge U. Press, New York (2010).

‣ J. S. Wettlaufer, "Accretion in Protoplanetary Disks by Collisional Fusion," *Astrophys. J.* **719**, 540 (2010).