PRACTICE PROBLEM SET 5

- **Chap 5**
- Questions are either directly from the text or a small variation of a problem in the text.
- Collaboration is okay, but final submission must be written individually. Mention all collaborators on your submission.
- The terms in the bracket indicate the problem number from the text.

Section 9.2

1) (Prob 6, Pg 240) a) Let S be the spehere of center $\boldsymbol{x}$ and radius R. What is the surface area of $S \cap \{|\boldsymbol{x}| < \rho\}$, the portion of S that lies within the spehere of center 0 and radius ρ?

b) Solve the wave equation in three dimensions for $t > 0$ with the initial conditions $\phi(\boldsymbol{x}) = 0$, $\psi(\boldsymbol{x}) = A$ for $|\boldsymbol{x}| < \rho$, and $\psi(\boldsymbol{x}) = 0$ for $|\boldsymbol{x}| > \rho$, where A is a constant.

c) Let $|\boldsymbol{x}_0| < \rho$. Ride the wave along a light ray emanating from $(\boldsymbol{x}_0, 0)$. That is, look at $u(\boldsymbol{x}_0 + t\boldsymbol{v}, t)$, where $|\boldsymbol{v}| = c$. Prove that

$$t \cdot u(\boldsymbol{x}_0 + t\boldsymbol{v}, t) \text{converges as } t \to \infty\,.$$

2) (Prob 13, Pg 241) Solve the wave equation in the half-space $\{(x, y, z, t) : z > 0\}$with the Neumann condition $\frac{\partial u}{\partial z} = 0$ on $z = 0$ and with initial data $\phi(x, y, z) \equiv 0$ and general $\psi(x, y, z)$.

3) (Prob 16, Pg 241) Solve part b) for the same problem in 2 dimensions. Furthermore, compute $u(0, t)$ by computing the integral explicitly and compute the limit of $u(0, t)$ as $t \to \infty$.

Section 14.1

4) (Prob 5, Pg 389) Solve $u_t + u^2 u_x = 0$ with $u(x, 0) = 2 + x$

5) (Prob 10, Pg 389) Solve $u_t + uu_x = 0$ with initial conditions $u(x, 0) = 1$ for $x \le 0$, $1 - x$ for $0 \le x \le 1$ and 0 for $x \ge 1$. Solve for all $t \ge 0$, allowing for a shock wave.

Section 14.3

6) (Prob 4, Pg 400) Find the curve $y = u(x)$ that makes the integral $\int_0^1 \left((u')^2 + xu\right) dx$ sationary subject to the constraints $u(0) = 0$ and $u(1) = 1$.

7) (Prob 7, Pg 401) Show that there are an infinite number of functions that minimize the integral

$$\int_0^2 (y')^2 (1 + y')^2 \quad \text{subject to } y(0) = 1 \text{ and } y(2) = 0\,.$$

They are continuous functions with piecewise continuous first derivatives.

8) (Prob 11, Pg 401) If the action $A[u] = \iint \left(u_{xx}^2 - u_t^2\right) dxdt$, show that the Euler-Lagrange equation is the beam equation $u_{tt} + u_{xxxx} = 0$, the equation for a stiff rod.